3D számítógépes geometria 2

Lineáris algebra alapok – ujjgyakorlat megoldások

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Ujjgyakorlat 1(a)

Feladat:

$$\begin{bmatrix} 10 & 0 & 0 & 0 & -2 & 0 \\ 3 & 9 & 0 & 0 & 0 & 3 \\ 0 & 7 & 8 & 7 & 0 & 0 \\ 3 & 0 & 8 & 7 & 5 & 0 \\ 0 & 8 & 0 & 9 & 9 & 13 \\ 0 & 4 & 0 & 0 & 2 & -1 \end{bmatrix}$$

Megoldás:

$$\begin{aligned} \mathrm{row} &= [1, 3, 6, 9, 13, 17, 20] \\ \mathrm{j} &= [1, 5, 1, 2, 6, 2, 3, 4, 1, 3, 4, 5, 2, 4, 5, 6, 2, 5, 6] \\ v &= [10, -2, 3, 9, 3, 7, 8, 7, 3, 8, 7, 5, 8, 9, 9, 13, 4, 2, -1] \end{aligned}$$

Ujjgyakorlat 1(b)

Feladat:

$$\begin{bmatrix} 0 & 2 & 0 & 0 \\ 1 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \end{bmatrix}$$

Megoldás:

$$\begin{aligned} \text{row} &= [1, 2, 4, 4, 5] \\ \text{j} &= [2, 1, 3, 2] \\ v &= [2, 1, 5, 3] \end{aligned}$$

Ujjgyakorlat 1.1

- (a) Elég felszorozni a v vektort
- (b)

```
struct Sparse{T}
    n::Int
    m::Int
    r::Array{Int}
    j::Array{Int}
    v::Array{T}
end
```

- Megoldás ⇒
  (csak kellő energiabefektetés után kibogarászandó)

```
function sparseadd(a::Sparse, b::Sparse)
    n, m = a.n, a.m
    @assert b.n == n && b.m == m "Matrices should have the same dimensions"
    @assert eltype(a.v) == eltype(b.v) "Matrices should have the same element type"
    r, j, v = Array{Int}(undef, n+1), zeros(Int, 0), zeros(eltype(a.v), 0)
    k = 1
    for i = 1:n
        r[i] = k
        ka, kb = a.r[i], b.r[i]
        while ka < a.r[i+1] || kb < b.r[i+1]
            if kb >= b.r[i+1] || (ka < a.r[i+1] && a.j[ka] < b.j[kb])
                push!(j, a.j[ka])
                push!(v, a.v[ka])
                ka += 1
            elseif ka >= a.r[i+1] || a.j[ka] > b.j[kb]
                push!(j, b.j[kb])
                push!(v, b.v[kb])
                kb += 1
            else
                push!(j, a.j[ka])
                push!(v, a.v[ka] + b.v[kb])
                ka += 1; kb += 1
            end
            k += 1
        end
    end
    r[n+1] = k
    Sparse(n, m, r, j, v)
end

# julia> a = Sparse(4,4,[1,2,4,4,5],[2,1,3,2],[2,1,5,3])
# julia> b = Sparse(4,4,[1,3,4,5,6],[3,4,1,3,2],[1,3,2,4,1])
# julia> sparseadd(a,b)
```

Ujjgyakorlat 2

Feladat:

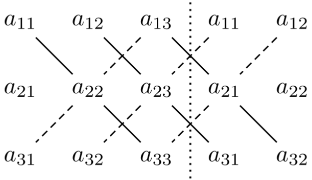
$$\begin{vmatrix} 2 & -6 & -1 \\ -3 & -1 & 7 \\ -8 & 1 & -2 \end{vmatrix} = ?$$

Megoldás:

$$\underbrace{2 \cdot (-1) \cdot (-2)}_{4} - \underbrace{2 \cdot 1 \cdot 7}_{14} - \underbrace{(-3) \cdot (-6) \cdot (-2)}_{-36} + \underbrace{(-3) \cdot 1 \cdot (-1)}_{3} + \underbrace{(-8) \cdot (-6) \cdot 7}_{336} - \underbrace{(-8) \cdot (-1) \cdot (-1)}_{-8}$$

$$4 - 14 - (-36) + 3 + 336 - (-8) = 373$$

Sarrus-szabály (123/231/312 pozitív, 132/213/321 negatív):



Ujjgyakorlat 3

Feladat:

$$\begin{aligned} 2x_2+5x_3 &= 1 \\ 2x_1+x_2+x_3 &= 1 \\ 3x_1+x_2 &= 2 \end{aligned}$$

Ujjgyakorlat 3

Feladat:

$$\begin{aligned} 2x_2+5x_3 &= 1 \\ 2x_1+\ x_2+\ x_3 &= 1 \\ 3x_1+\ x_2 \qquad &= 2 \end{aligned}$$

Megoldás:

$$\begin{aligned} 3x_1+\ x_2 \qquad &= 2 \\ 2x_2+5x_3 &= 1 \\ \frac{1}{3}x_2+\ x_3 &= -\frac{1}{3} \end{aligned}$$

Ujjgyakorlat 3

Feladat:

$$\begin{aligned} 2x_2 + 5x_3 &= 1 \\ 2x_1 + x_2 + x_3 &= 1 \\ 3x_1 + x_2 \phantom{{}+x_3} &= 2 \end{aligned}$$

Megoldás:

$$\begin{aligned} 3x_1 + x_2 \phantom{{}+5x_3} &= 2 \\ 2x_2 + 5x_3 &= 1 \\ \frac{1}{3}x_2 + x_3 &= -\frac{1}{3} \end{aligned}$$

Pivotáltuk a harmadik sort. Folytatva:

$$\begin{aligned} 3x_1 + x_2 \phantom{{}+5x_3} &= 2 \\ 2x_2 + 5x_3 &= 1 \\ \frac{1}{6}x_3 &= -\frac{1}{2} \end{aligned}$$

$\Rightarrow x_3 = -3 \ \Rightarrow x_2 = 8 \ \Rightarrow x_1 = -2$

Ujjgyakorlat 4

Feladat:

$$\begin{aligned} 3x_1 - 2x_2 &= 0 \\ -x_1 + 2x_2 &= 4 \end{aligned}$$

Ujjgyakorlat 4

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Megoldás:

$x_1^{(1)} = 2 \cdot 0/3 = 0$
$x_2^{(1)} = (4 + 0)/2 = 2$

Ujjgyakorlat 4

Feladat:

$$3x_1 - 2x_2 = 0$$
$$-x_1 + 2x_2 = 4$$

Megoldás:

$x_1^{(1)} = 2 \cdot 0/3 = 0$
$x_2^{(1)} = (4+0)/2 = 2$
$x_1^{(2)} = 2 \cdot 2/3 = \frac{4}{3}$
$x_2^{(2)} = (4+\frac{4}{3})/2 = \frac{8}{3}$

Ujjgyakorlat 4

Feladat:

$$\begin{aligned} 3x_1 - 2x_2 &= 0 \\ -x_1 + 2x_2 &= 4 \end{aligned}$$

Megoldás:

$x_1^{(1)} = 2 \cdot 0/3 = 0$
$x_2^{(1)} = (4 + 0)/2 = 2$
$x_1^{(2)} = 2 \cdot 2/3 = \frac{4}{3}$
$x_2^{(2)} = (4 + \frac{4}{3})/2 = \frac{8}{3}$
$x_1^{(3)} = 2 \cdot \frac{8}{3}/3 = \frac{16}{9}$
$x_2^{(3)} = (4 + \frac{16}{9})/2 = \frac{26}{9}$

Ujjgyakorlat 4

Feladat:

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Megoldás:

$x_1^{(1)} = 2 \cdot 0/3 = 0$
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$x_2^{(3)} = (4 + \frac{16}{9})/2 = \frac{26}{9}$
$x_1^{(4)} = 2 \cdot \frac{26}{9}/3 = \frac{52}{27} = 1\frac{25}{27} \approx 2$ ($< 5\%$ relatív hiba)
$x_2^{(4)} = (4 + \frac{52}{27})/2 = \frac{80}{27} = 2\frac{26}{27} \approx 3$ ($< 2\%$ relatív hiba)

Ujjgyakorlat 4

Feladat:

$$3x_1 - 2x_2 = 0$$
$$-x_1 + 2x_2 = 4$$

Megoldás:

$x_1^{(1)} = 2 \cdot 0/3 = 0$
$x_2^{(1)} = (4+0)/2 = 2$
$x_1^{(2)} = 2 \cdot 2/3 = \frac{4}{3}$
$x_2^{(2)} = (4+\frac{4}{3})/2 = \frac{8}{3}$
$x_1^{(3)} = 2 \cdot \frac{8}{3}/3 = \frac{16}{9}$
$x_2^{(3)} = (4+\frac{16}{9})/2 = \frac{26}{9}$
$x_1^{(4)} = 2 \cdot \frac{26}{9}/3 = \frac{52}{27} = 1\frac{25}{27} \approx 2$ ($<5\%$ relatív hiba)
$x_2^{(4)} = (4+\frac{52}{27})/2 = \frac{80}{27} = 2\frac{26}{27} \approx 3$ ($<2\%$ relatív hiba)

- További iterációk:
  - $x_1^{(10)} = 1.999...$, $x_2^{(10)} = 2.999...$ (3 tizedesjegyig OK)
  - $x_1^{(35)} = 2$, $x_2^{(35)} = 3$ (64 bites float pontosságával)

Ujjgyakorlat 5(a)

Feladat:

$$\begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$$

Megoldás:

Ujjgyakorlat 5(a)

Feladat:

$$\begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$$

Megoldás:

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \Rightarrow$$

Ujjgyakorlat 5(a)

Feladat:

$$\begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$$

Megoldás:

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix} = 4 \begin{bmatrix} 1/4 \\ 1/4 \\ 1 \end{bmatrix} \Rightarrow$$

Ujjgyakorlat 5(a)

Feladat:

$$\begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$$

Megoldás:

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix} = 4 \begin{bmatrix} 1/4 \\ 1/4 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 5/2 \\ 5/2 \\ 4 \end{bmatrix} = 4 \begin{bmatrix} 5/8 \\ 5/8 \\ 1 \end{bmatrix} \Rightarrow$$

Ujjgyakorlat 5(a)

Feladat:

$$\begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$$

Megoldás:

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix} = 4 \begin{bmatrix} 1/4 \\ 1/4 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 5/2 \\ 5/2 \\ 4 \end{bmatrix} = 4 \begin{bmatrix} 5/8 \\ 5/8 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 7/4 \\ 7/4 \\ 4 \end{bmatrix} = 4 \begin{bmatrix} 7/16 \\ 7/16 \\ 1 \end{bmatrix}$$

Ujjgyakorlat 5(a)

Feladat:

$$\begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$$

Megoldás:

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix} = 4 \begin{bmatrix} 1/4 \\ 1/4 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 5/2 \\ 5/2 \\ 4 \end{bmatrix} = 4 \begin{bmatrix} 5/8 \\ 5/8 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 7/4 \\ 7/4 \\ 4 \end{bmatrix} = 4 \begin{bmatrix} 7/16 \\ 7/16 \\ 1 \end{bmatrix}$$

Pontos megoldás:

$$\lambda = 4, \quad v = \begin{bmatrix} 1/2 \\ 1/2 \\ 1 \end{bmatrix}$$

Ujjgyakorlat 5(b)

Feladat:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Megoldás:

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \Rightarrow$$

Ujjgyakorlat 5(b)

Feladat:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Megoldás:

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 6 \\ 15 \\ 24 \end{bmatrix} = 24 \begin{bmatrix} 1/4 \\ 5/8 \\ 1 \end{bmatrix} \Rightarrow$$

Ujjgyakorlat 5(b)

Feladat:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Megoldás:

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 6 \\ 15 \\ 24 \end{bmatrix} = 24 \begin{bmatrix} 1/4 \\ 5/8 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 9/2 \\ 81/8 \\ 63/4 \end{bmatrix} = \frac{63}{4} \begin{bmatrix} 2/7 \\ 9/14 \\ 1 \end{bmatrix} \Rightarrow$$

Ujjgyakorlat 5(b)

Feladat:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Megoldás:

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 6 \\ 15 \\ 24 \end{bmatrix} = 24 \begin{bmatrix} 1/4 \\ 5/8 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 9/2 \\ 81/8 \\ 63/4 \end{bmatrix} = \frac{63}{4} \begin{bmatrix} 2/7 \\ 9/14 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 32/7 \\ 145/14 \\ 113/7 \end{bmatrix} \approx 16.14 \begin{bmatrix} 0.2832 \\ 0.6416 \\ 1 \end{bmatrix}$$

Ujjgyakorlat 5(b)

Feladat:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Megoldás:

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 6 \\ 15 \\ 24 \end{bmatrix} = 24 \begin{bmatrix} 1/4 \\ 5/8 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 9/2 \\ 81/8 \\ 63/4 \end{bmatrix} = \frac{63}{4} \begin{bmatrix} 2/7 \\ 9/14 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 32/7 \\ 145/14 \\ 113/7 \end{bmatrix} \approx 16.14 \begin{bmatrix} 0.2832 \\ 0.6416 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 6 \\ 15 \\ 24 \end{bmatrix} = 24 \begin{bmatrix} 2/8 \\ 5/8 \\ 8/8 \end{bmatrix} \Rightarrow \begin{bmatrix} 36/8 \\ 81/8 \\ 126/8 \end{bmatrix} = \frac{63}{4} \begin{bmatrix} 4/14 \\ 9/14 \\ 14/14 \end{bmatrix} \Rightarrow \begin{bmatrix} 64/14 \\ 145/14 \\ 226/14 \end{bmatrix} = \frac{113}{7} \begin{bmatrix} 64/226 \\ 145/226 \\ 226/226 \end{bmatrix}$$

Pontos megoldás:

$$\lambda = \frac{15 + \sqrt{297}}{2} \approx 16.12, \quad v \approx \begin{bmatrix} 0.2834 \\ 0.6417 \\ 1 \end{bmatrix}$$