Labeled Trimmed Fitting

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Outline

1. Introduction
2. Labeling
3. Labeled Parameterization and Extension
   - Parameterization
   - Extension
   - Fitting Results
4. Automatic Labeling
   - Admissible Configurations
   - Algorithm
   - Labeling Results
5. Conclusions

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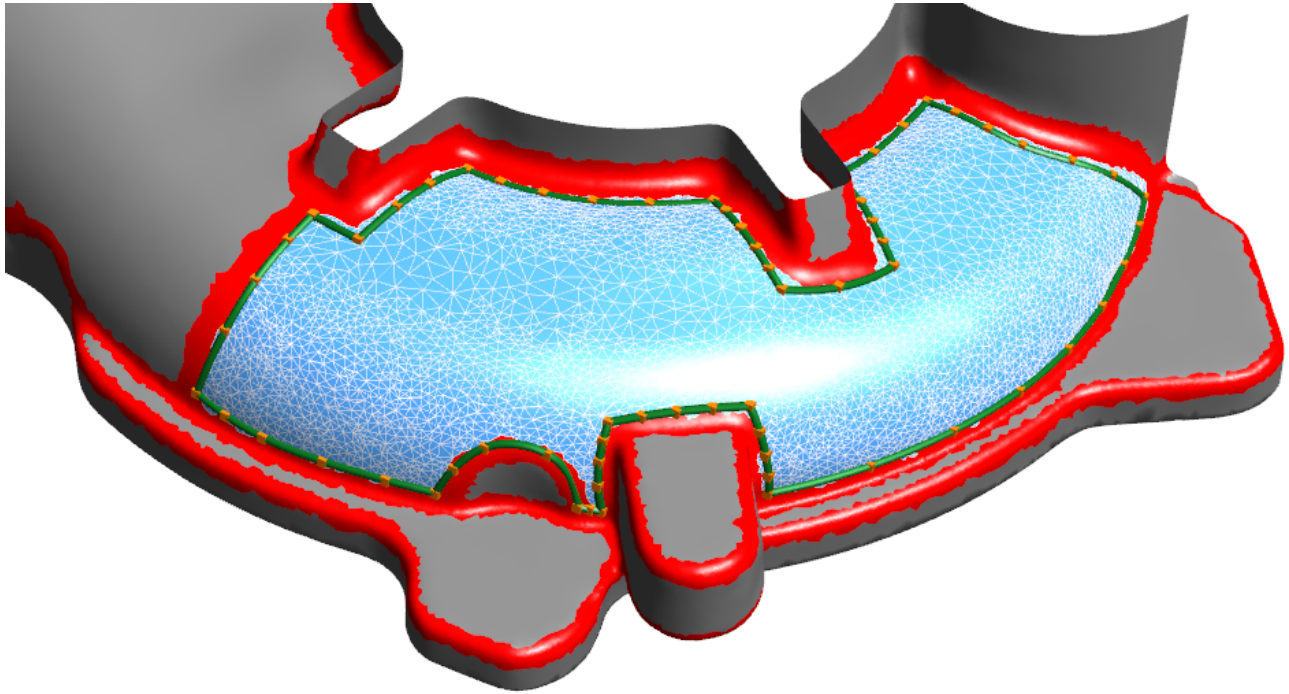
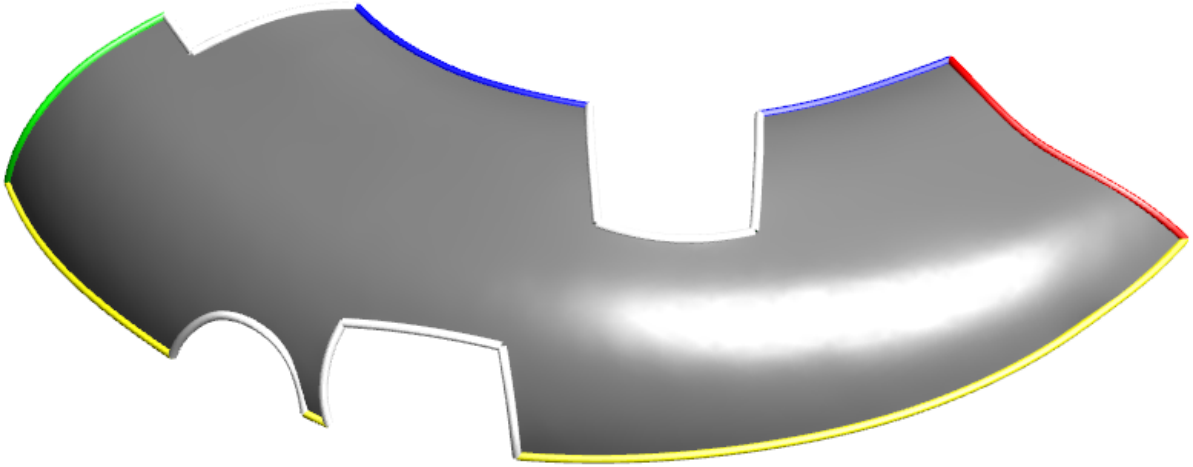
Context: Reverse Engineering of CAD Models



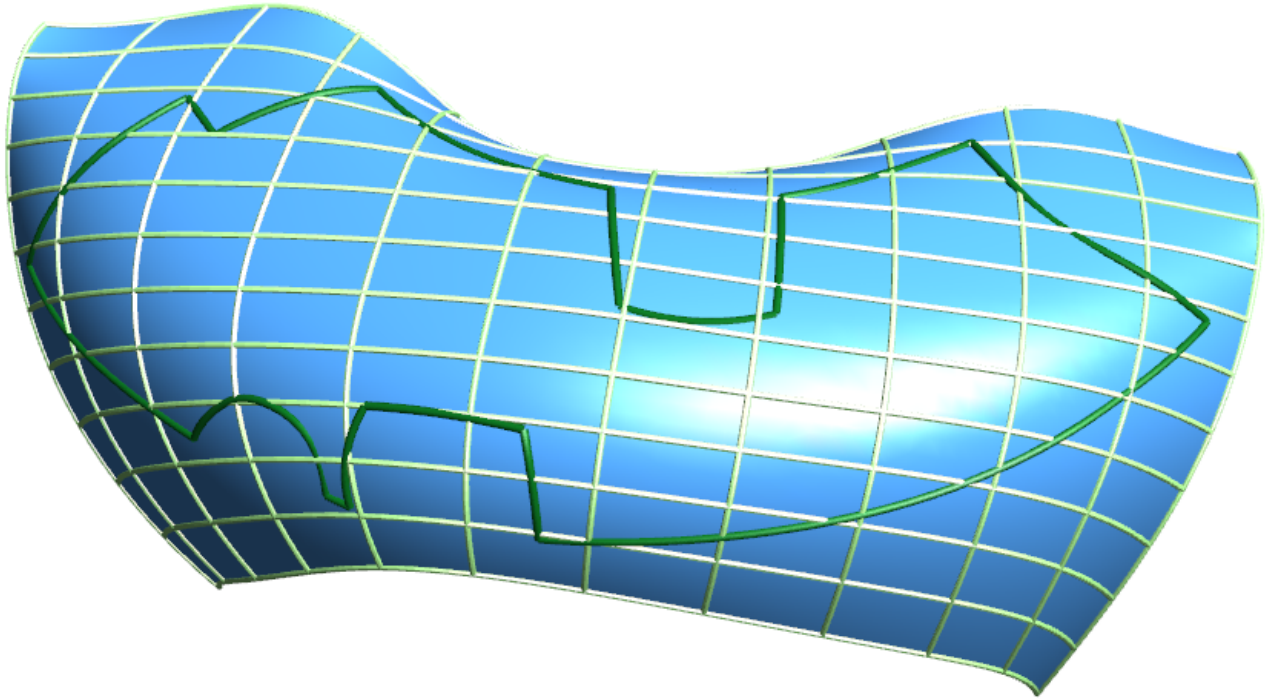
Segmented CAD Model

Context: Reverse Engineering of CAD Models



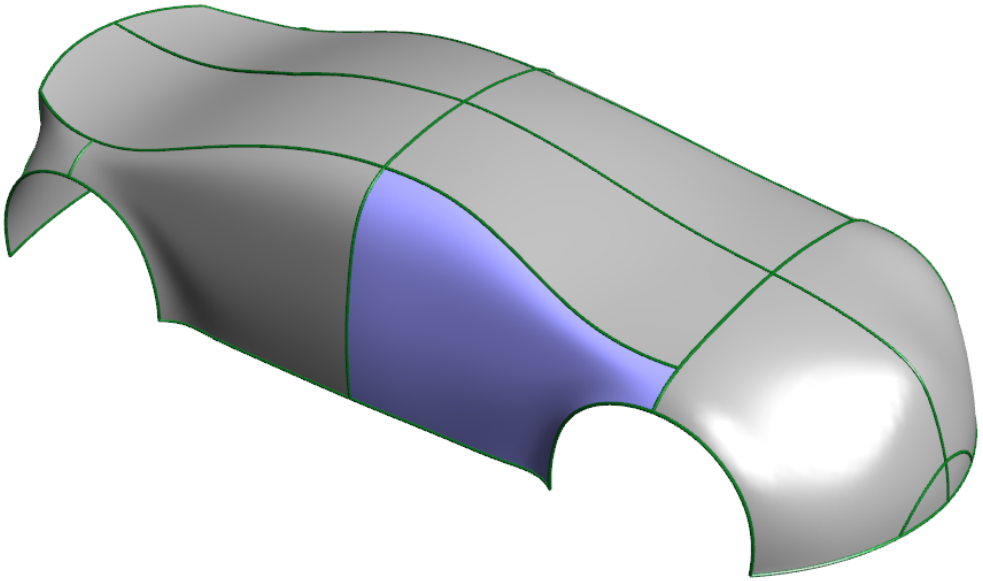
Patch classified as free-form

Context: Reverse Engineering of CAD Models

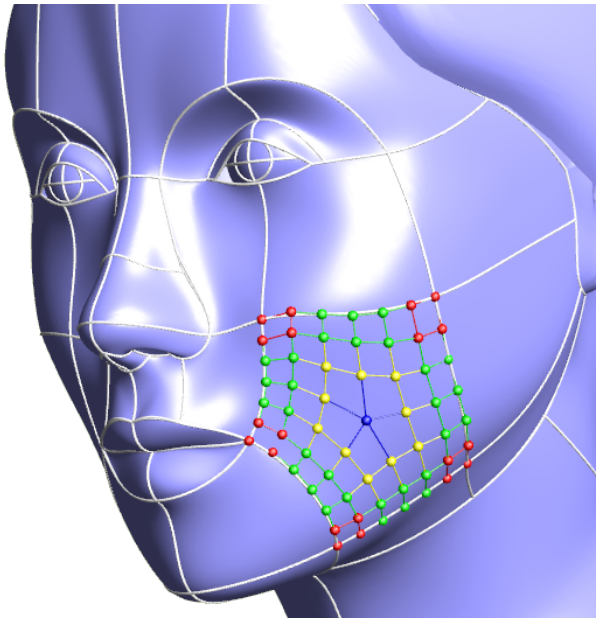


Trimmed Tensor-product B-Spline fit

Context: Conversion of Non-standard Representations

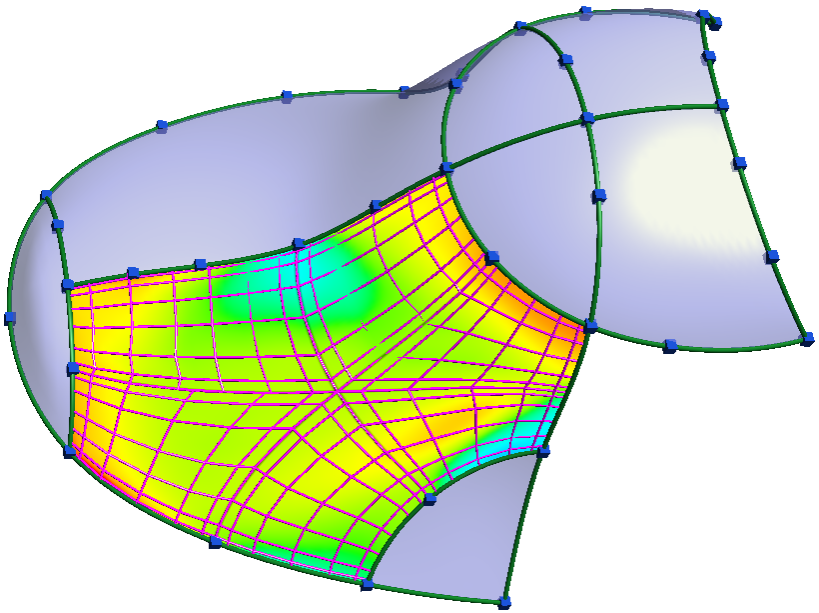


Curve network interpolation

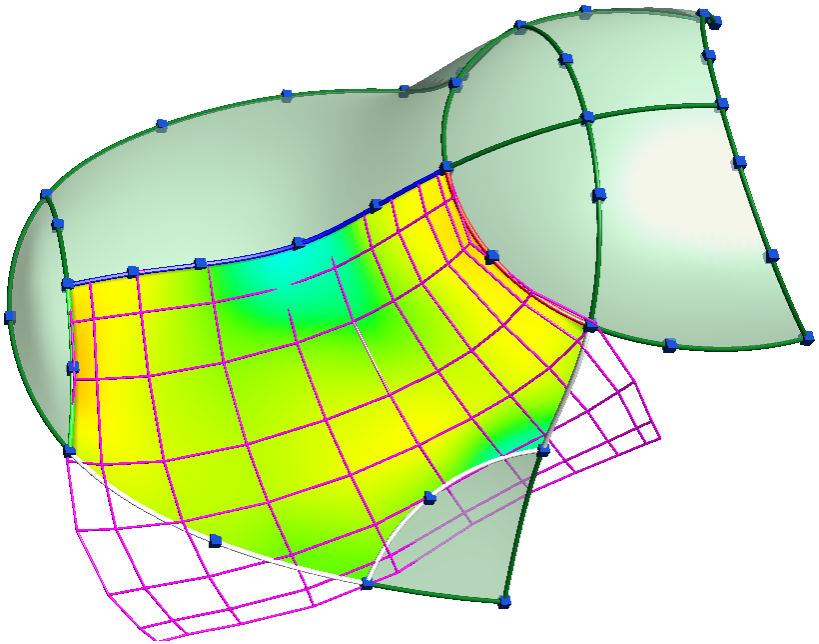


Generalized Bézier

Representing Multi-Sided Regions



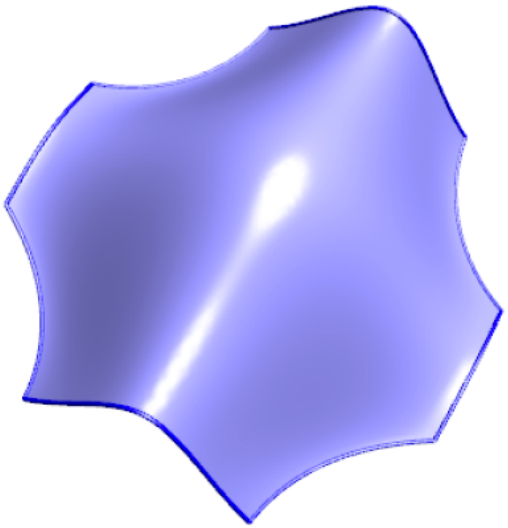
Quadrilateral Split

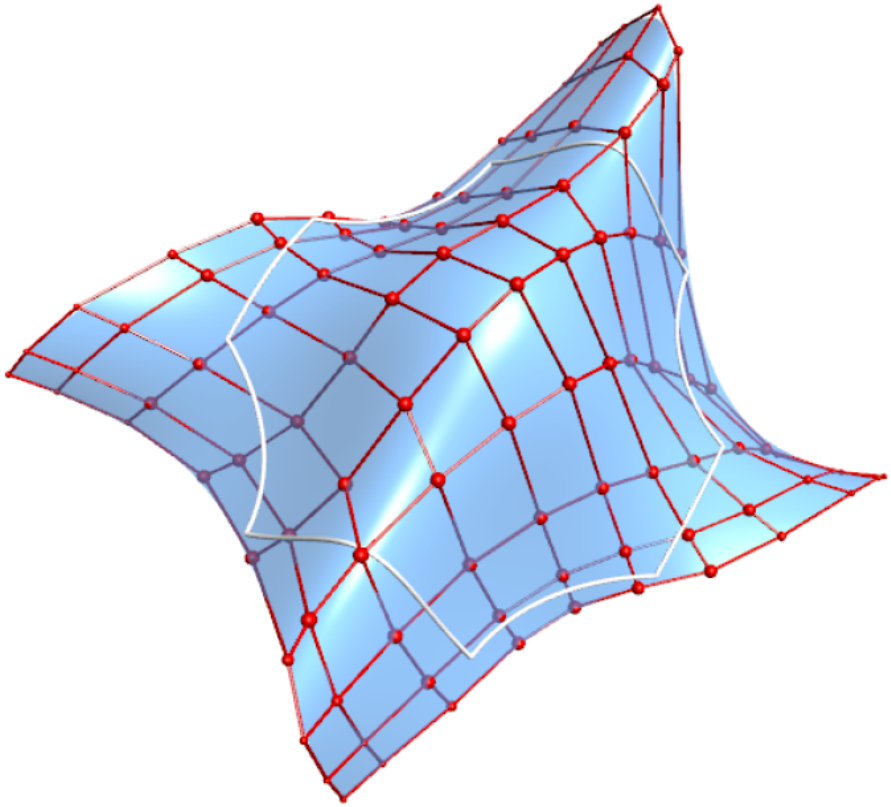


Trimmed Patch

Goal: Fitting triangle meshes w/ *trimmed* tensor-product B-splines

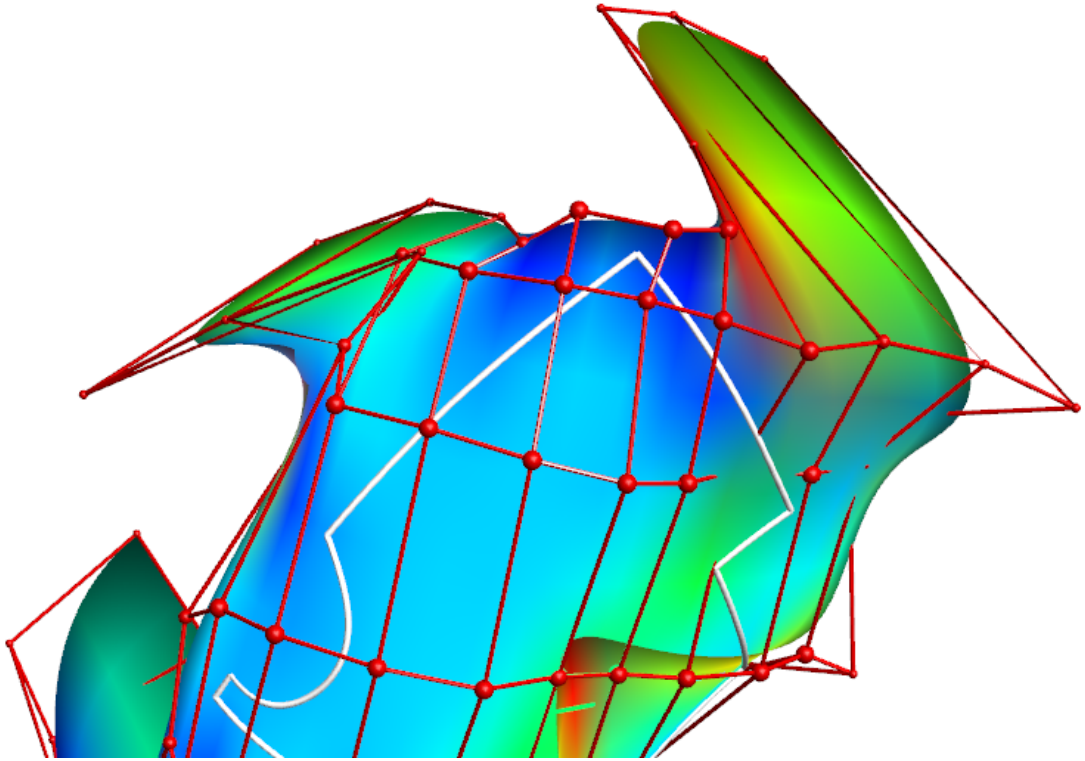
Issues of Trimmed Fitting





Geometry Alignment

Issues of Trimmed Fitting



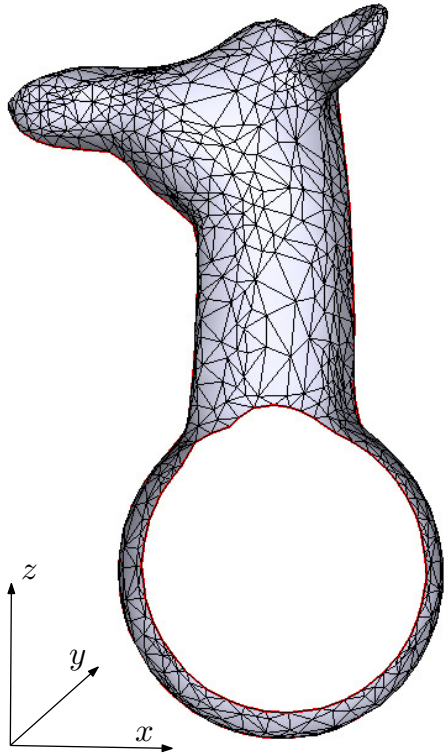
"Weak" control points!

Tensor product fitting

- Many methods – most common: iterative least-squares + regularization (thin-plate energy, etc.)
- Objectives:
  - Fitting error within tolerance
  - "Fairness" (curvature distribution, isophotes, etc.)
  - Small number of control points
  - Small trimmed region and controlled oscillation ("weak" control points)
- Initial parameterization of data points – fundamentally determines fit quality

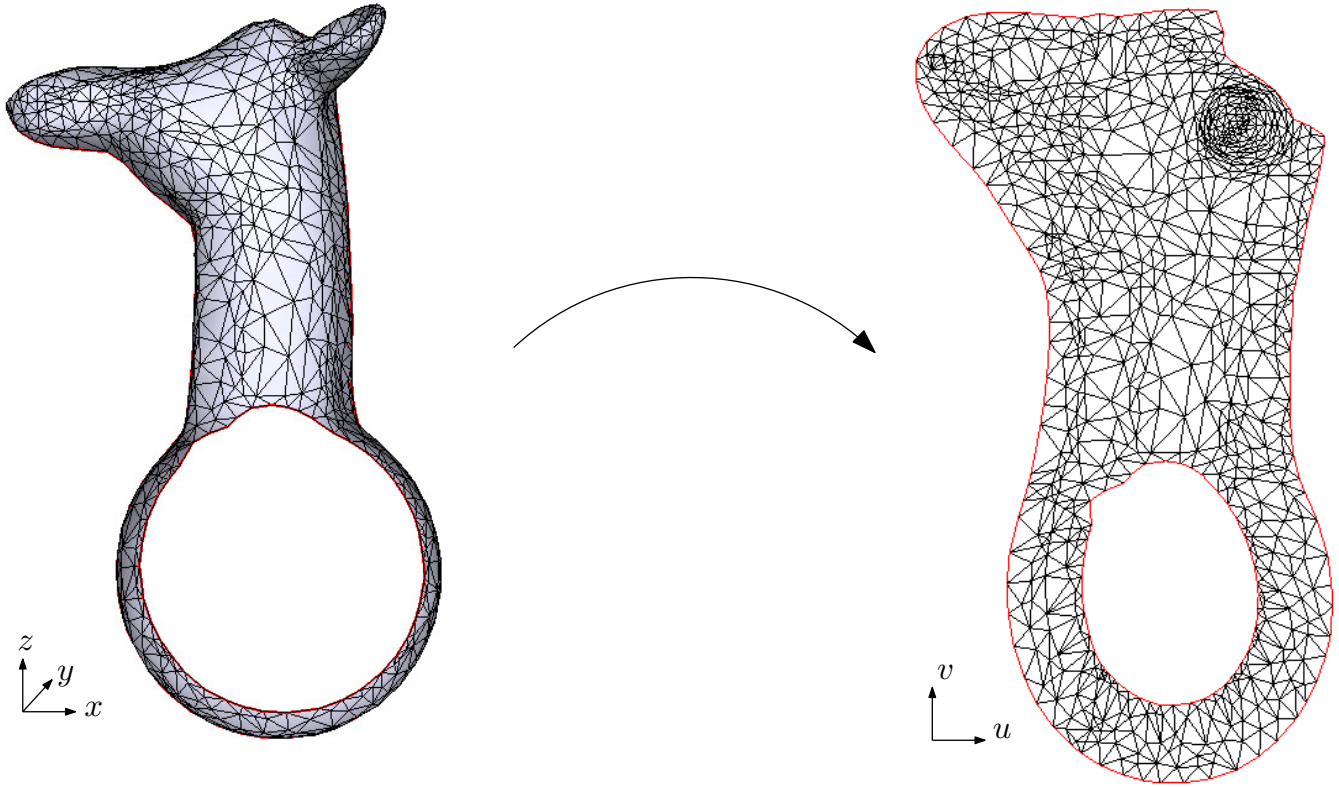
Parameterization - Task: Flattening

3D Triangle Mesh



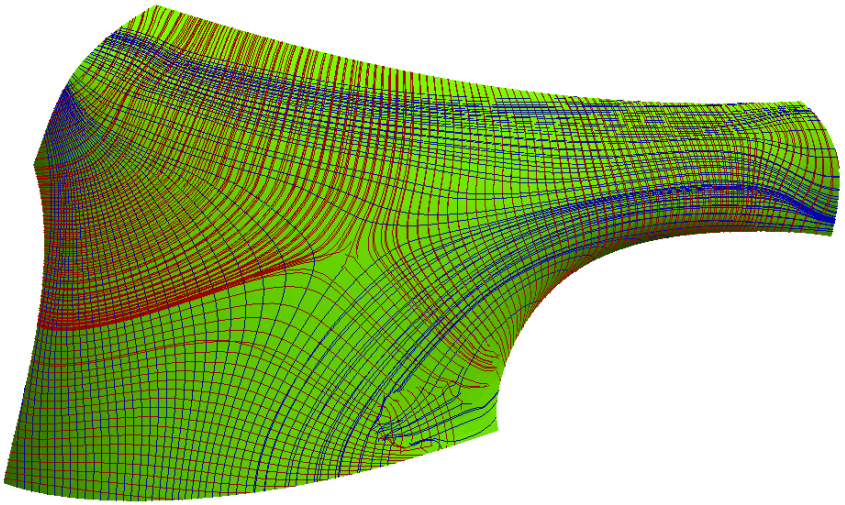
Parameterization - Task: Flattening

3D Triangle Mesh $\longrightarrow$ 2D Plane



Parameterization for Fitting

- Known parameterization methods
  - Projecting on PCA plane – for simple geometries
  - Projecting on Coons patch, etc. – need untrimmed boundary
  - Distortion minimization, curvature alignment – not crucial for trimmed fitting



- No simple characterization of parameterization quality for fitting
- Our approach: develop useful heuristics

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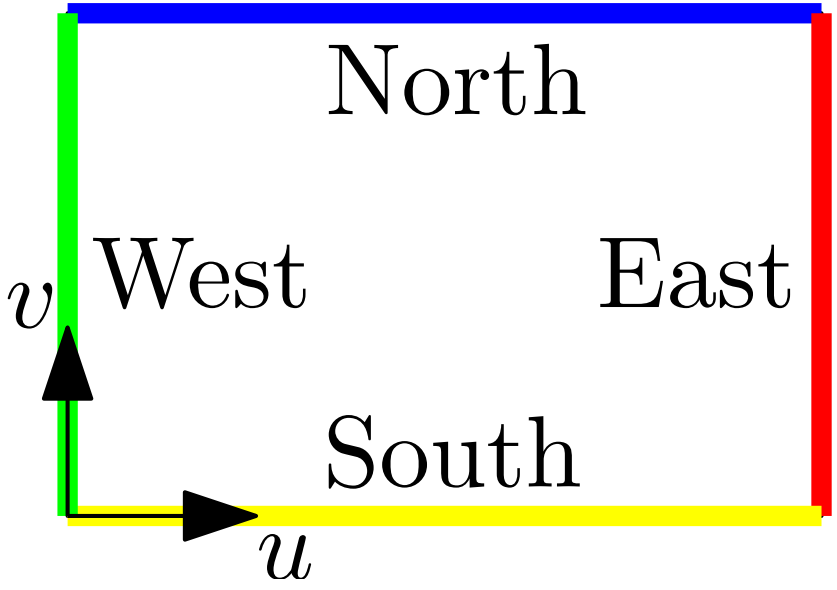
The Central Idea: Labeling

- Tensor-product – domain rectangle
- Labeling of boundary segments:
  - North
  - West
  - South
  - East
  - Unlabeled (trim curve)
- Labels can be multi-segment
- Labels can be unassigned



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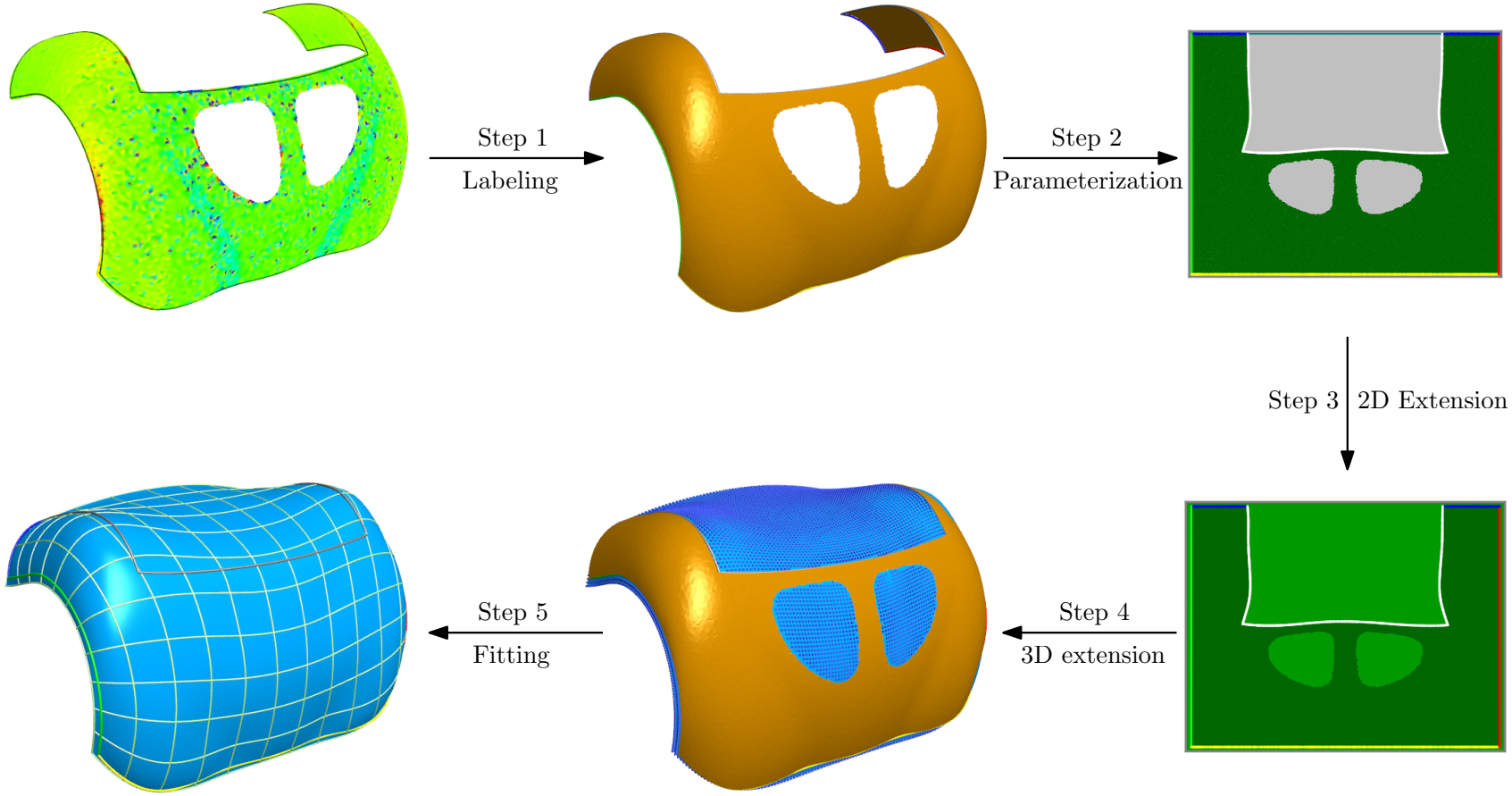
Parameterizing and extending trimmed regions for tensor-product surface fitting

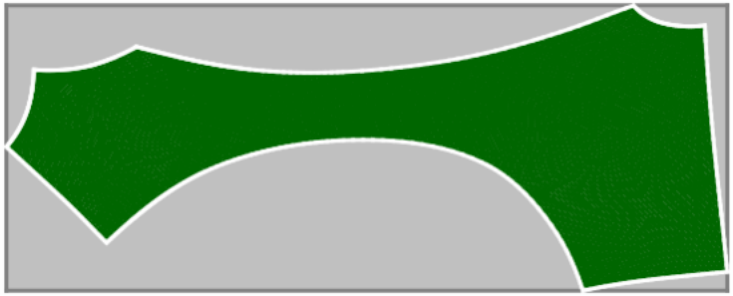
M. Vaitkus, T. Várady

Computer-Aided Design, 104, 125–140 (2018)
https://doi.org/10.1016/j.cad.2017.11.008

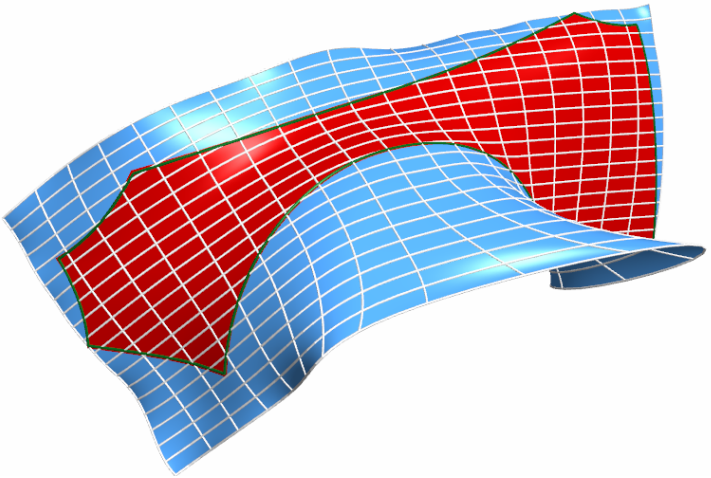
Labeled Parameterization and Extension – Workflow



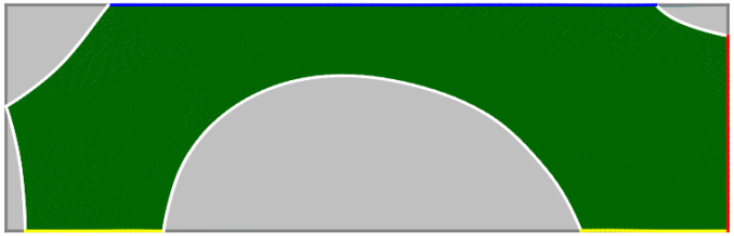
Labeled Parameterization and Extension



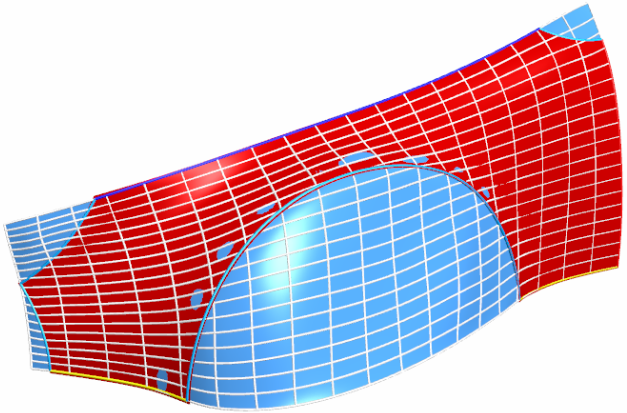
Parameterization w/o labeling



Fit w/o labeling



Parameterization w/ labeling



Fit w/ labeling

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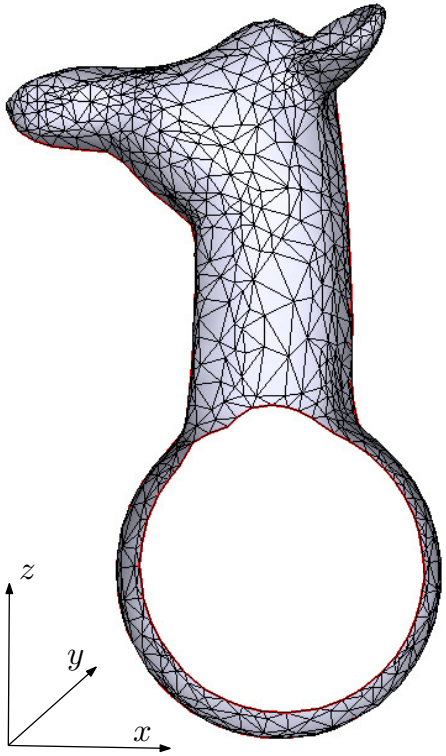
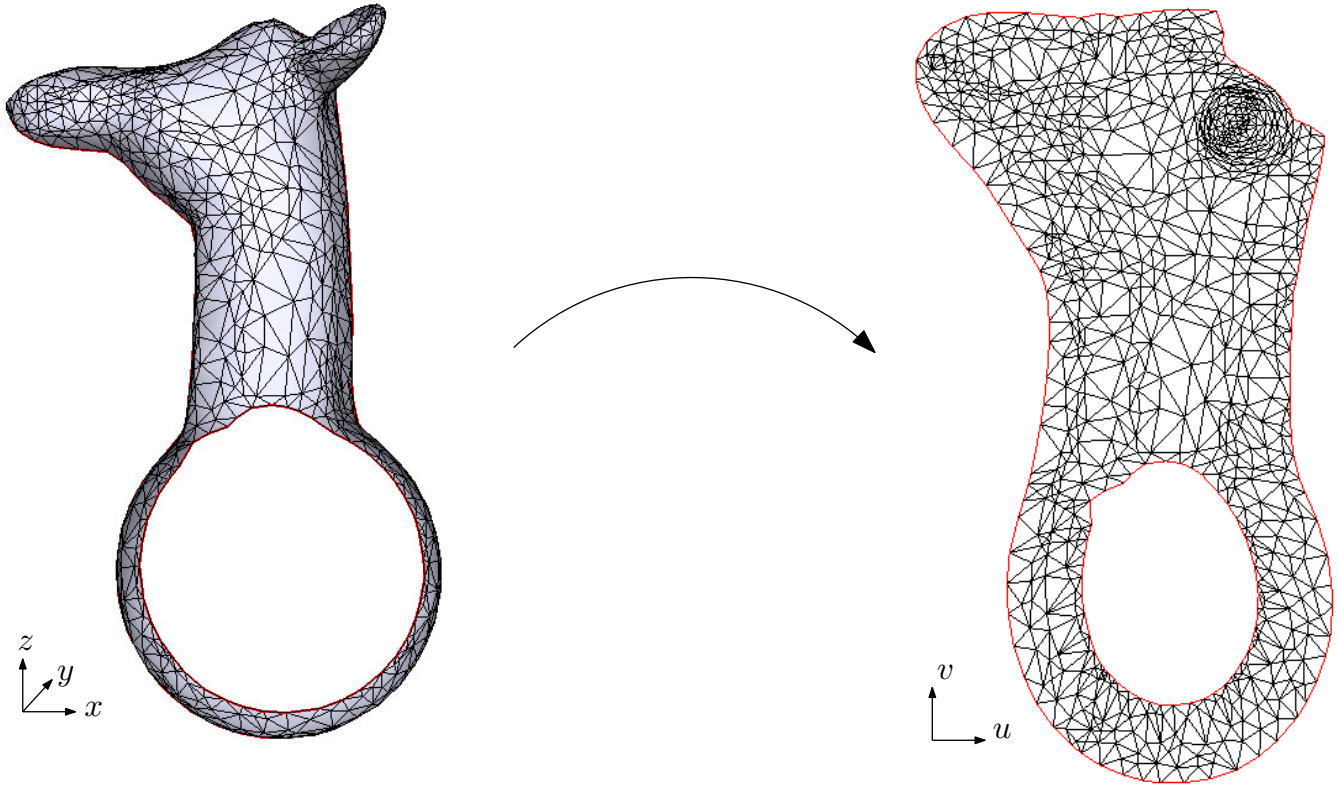
Parameterization - Task: Flattening

3D Triangle Mesh

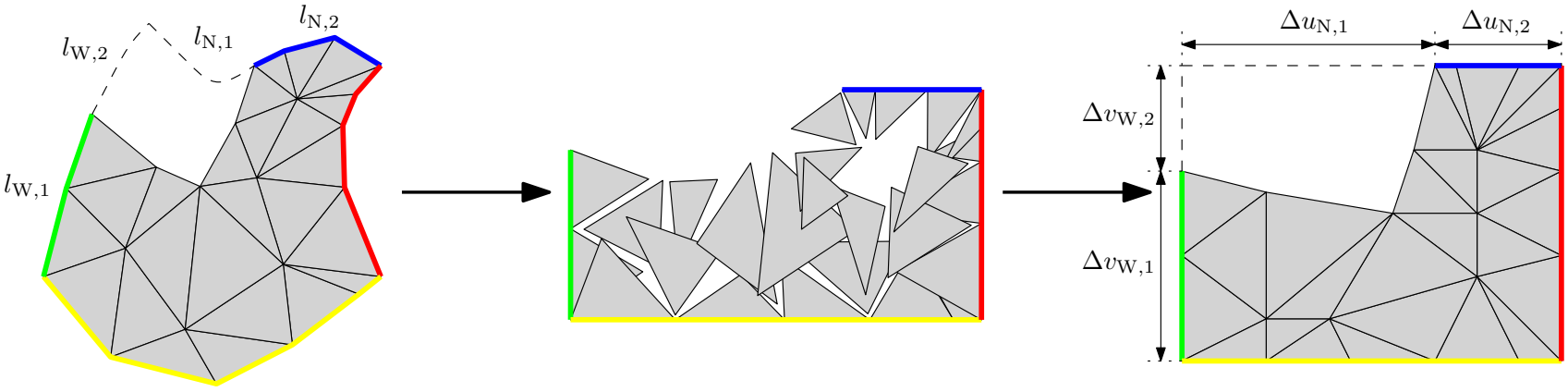


Parameterization - Task: Flattening

3D Triangle Mesh $\longrightarrow$ 2D Plane



Computing a Labeled Parameterization



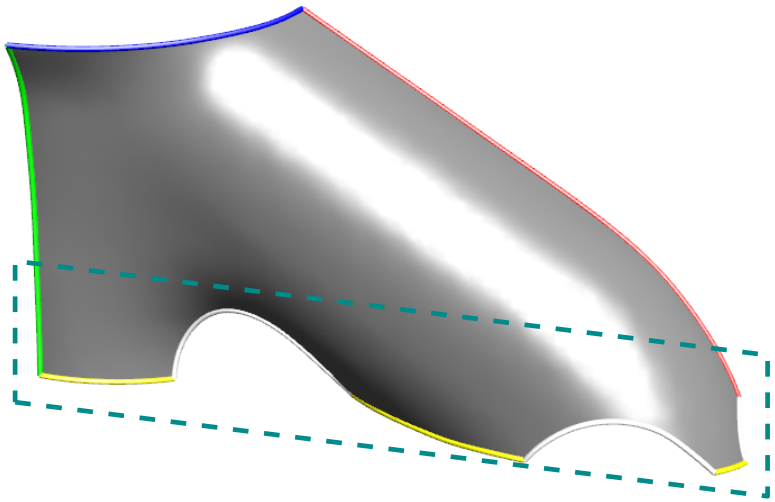
3D mesh with labels and guiding frame

Triangles mapped isometrically to the domain

Consistent 2D Mesh

- Constrained minimization of As-Rigid-As-Possible distortion energy
- First enforce label orientation, then positions
- Quadratic energies, linear constraints – solve 2 sparse linear systems

The Need for Guiding Frames



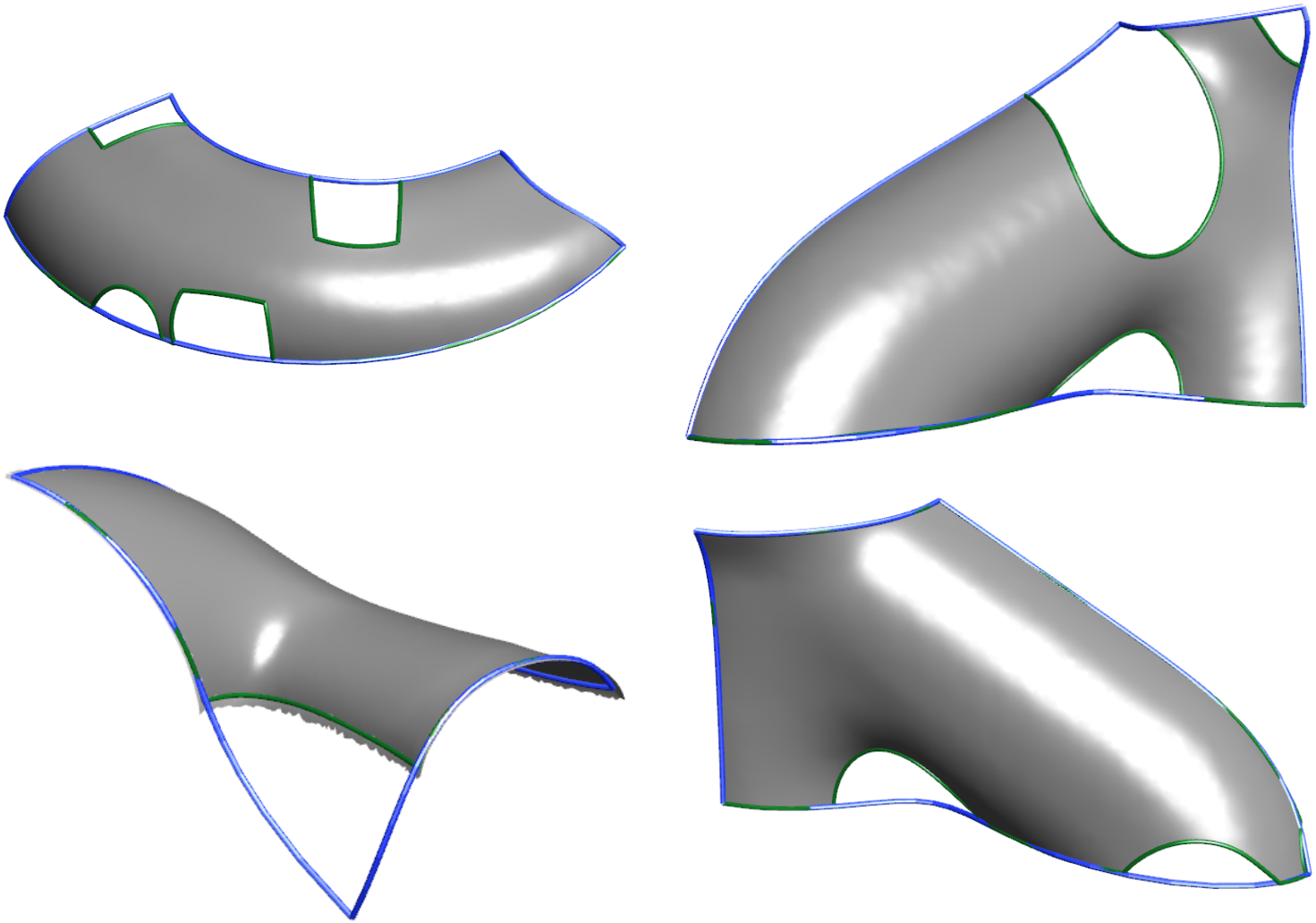
w/o frame constraints

w/ frame constraints

Computing a Labeled Parameterization: Guiding Frame

- Need to set length ratios along boundaries $\longrightarrow$ "guess" TP boundary loop?
- Iteratively fit a frame of ≤ 4 Bézier curves to labeled boundaries – "Guiding Frame"
- Infer boundary ratios from Guiding Frame

Guiding Frame – Examples



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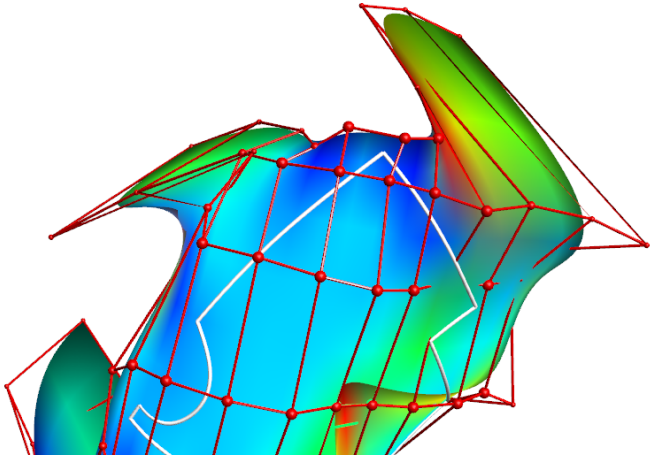
Extension

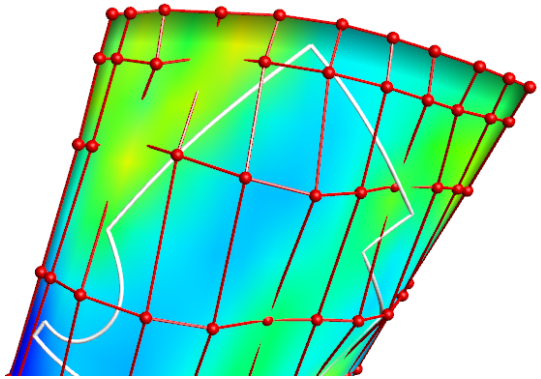
- Extend in 3D to avoid weak control points
- Add new points $u - v$ plane and triangulate
- Pull back to 3D by minimization of energy:

$$E_{\mathrm{3D}} = (1 - w_{\mathrm{Fair}})E_{\mathrm{Ext.}} + w_{\mathrm{Fair}}E_{\mathrm{Fair}},$$

- $E_{\mathrm{Ext.}}$: enforces smooth extension
- E_{Fair}: enforces geometry fairness



Fit without extension



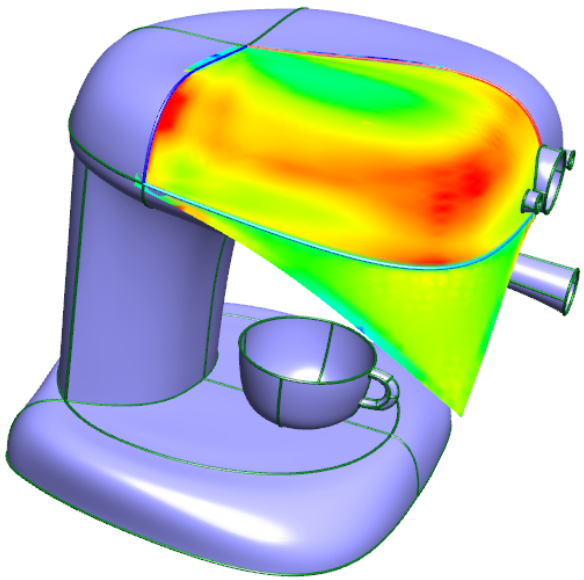
Fit with extension

Extension energy

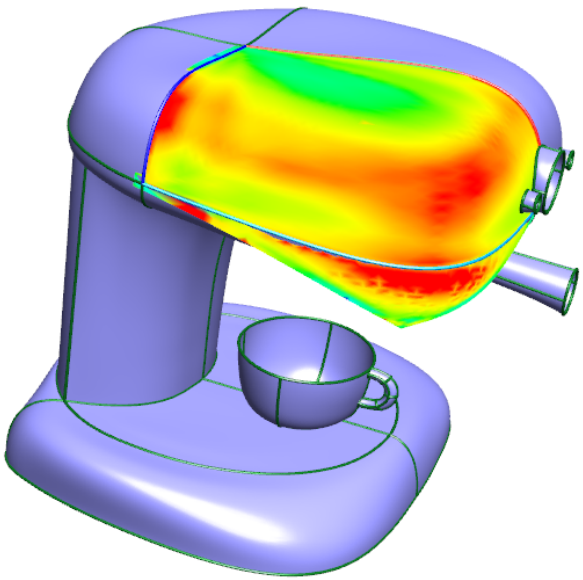
- Polyharmonic energy:

$$E_{\text{Ext.}} = \left\| \nabla^k \mathbf{p} \right\|^2$$

- Enforces G^{k-1} connection through boundary



$k = 2$



$k = 3$

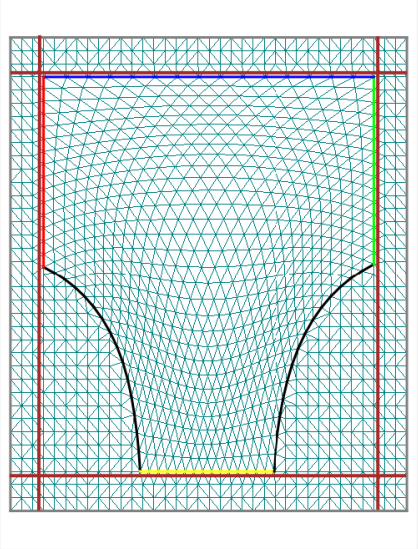
Fairness energy

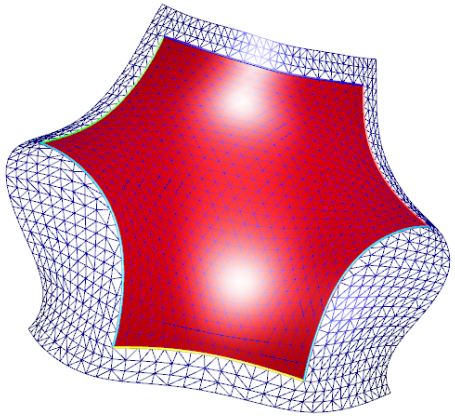
- Fairness energy:

$$E_{\text{Fair.}} = (1 - w_{\text{Iso}})E_{\text{Mesh}} + w_{\text{Iso}}E_{\text{Iso}},$$

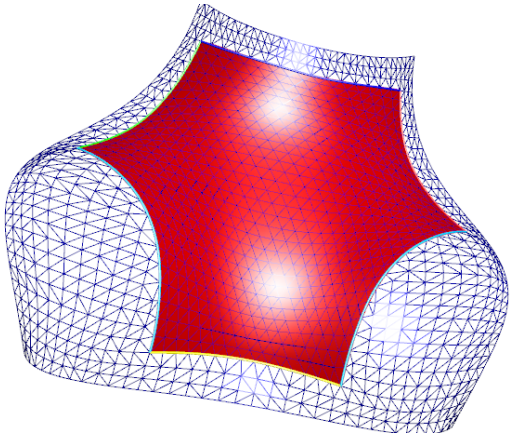
- E_{Mesh}: 3D ARAP distortion
- E_{Iso}: Isocurve fairness:

$$E_{\text{Iso}} = \int_{c \in C} \left(\left| \frac{d^k x(t)}{dt^k} \right|^2 + \left| \frac{d^k y(t)}{dt^k} \right|^2 + \left| \frac{d^k z(t)}{dt^k} \right|^2 \right) dt$$





Without fairing



With fairing

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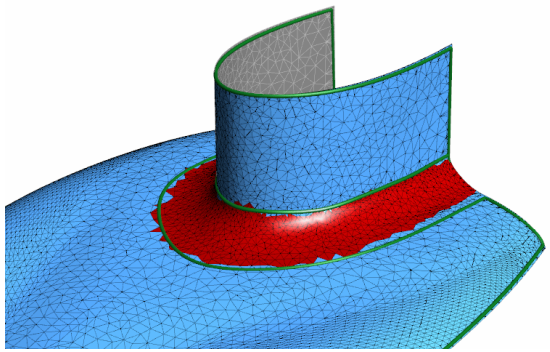
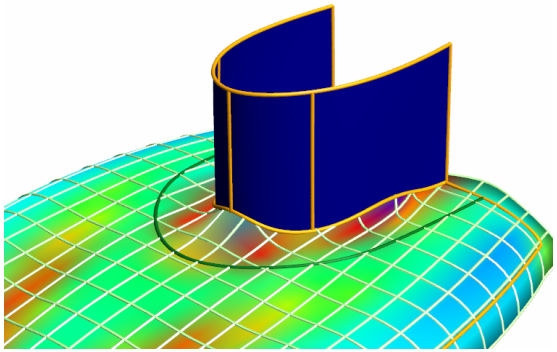
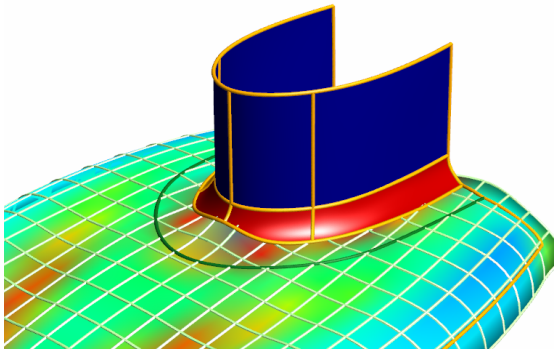
Results



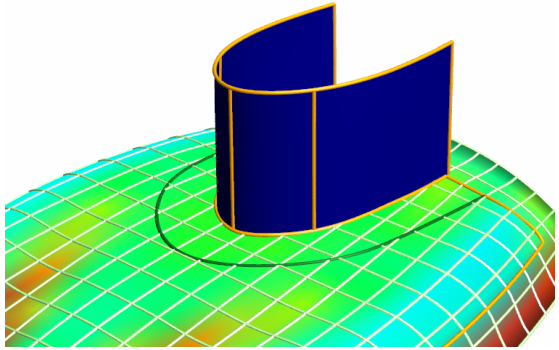
Mesh



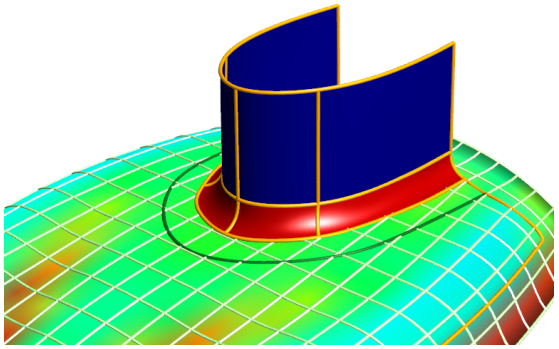
Inters. w/o extension



Blend w/o extension

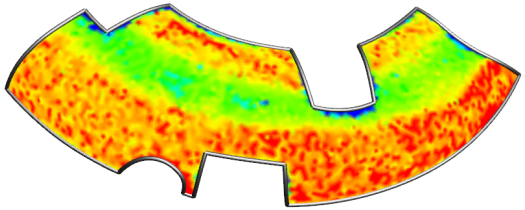


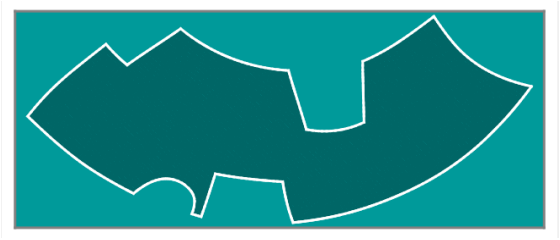
Inters. w/ extension



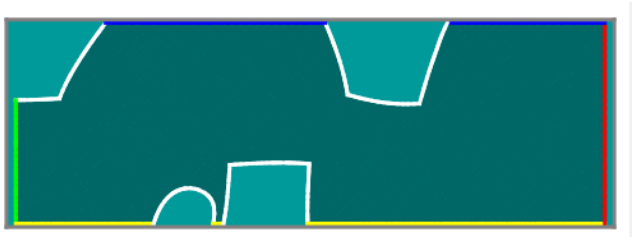
Blend w/ extension

Results

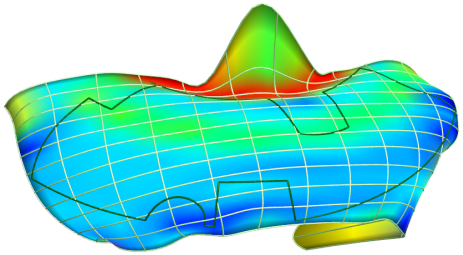




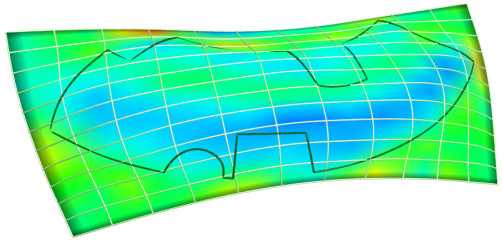
ARAP w/o labeling



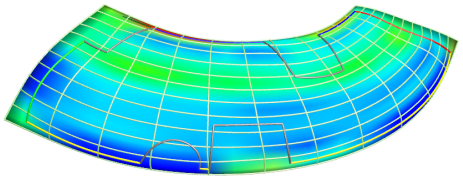
ARAP w/ labeling



Default

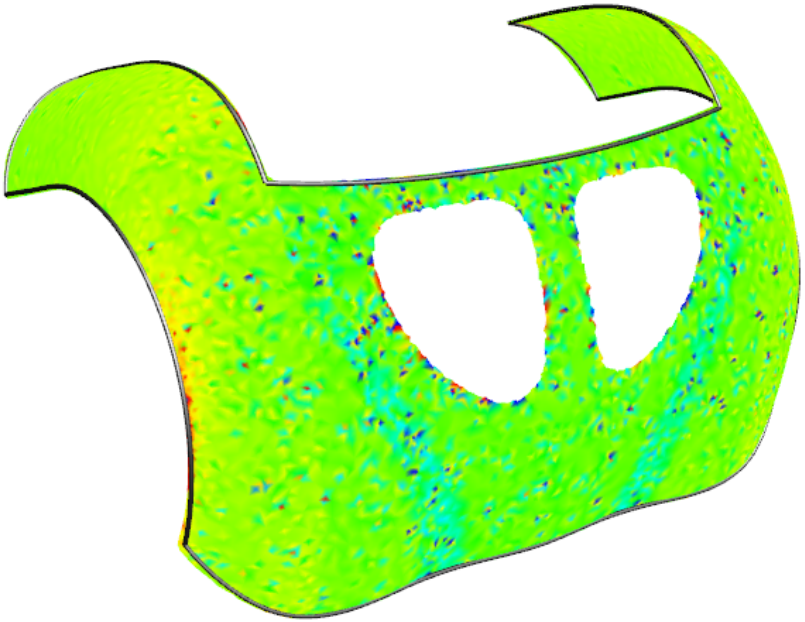


Extension



Labeling & Extension

Results



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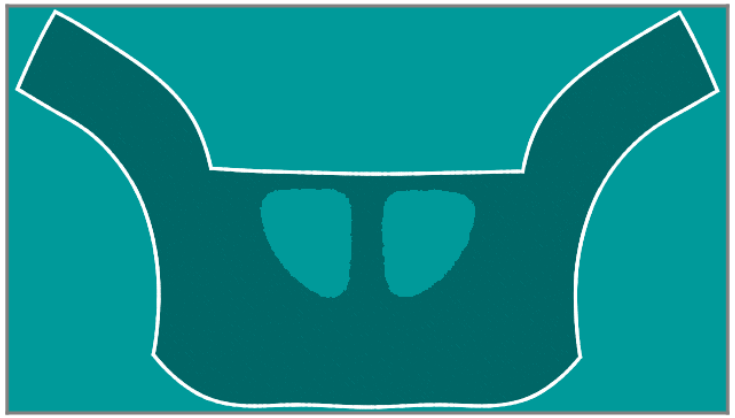

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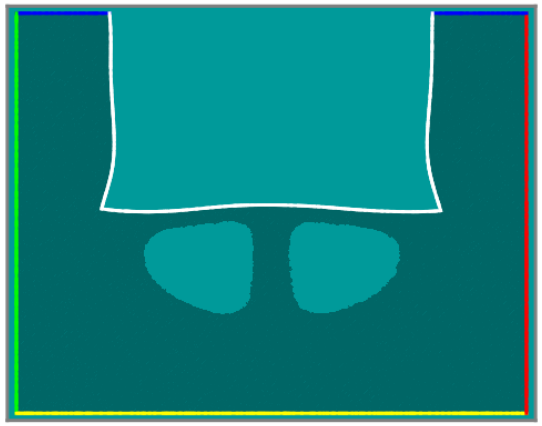
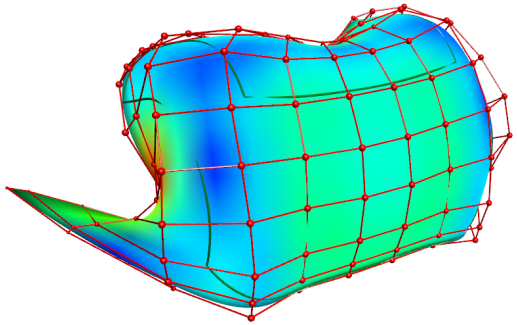
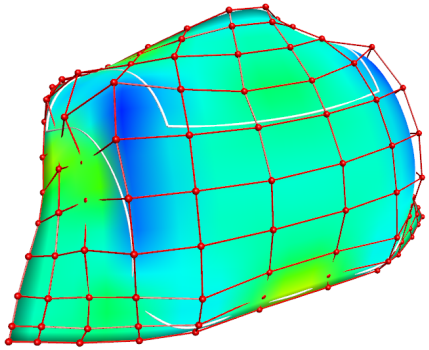


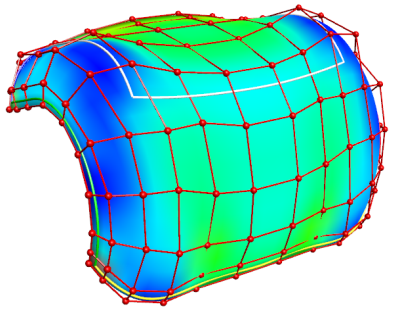
ARAP w/ labeling



Default



Extension



Labeling + Extension

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A Labeling Algorithm for Trimmed Surface Fitting

M. Vaitkus, T. Várady

Accepted to Computer-Aided Design and Applications, 2018

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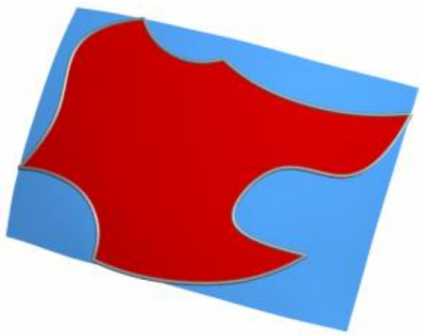
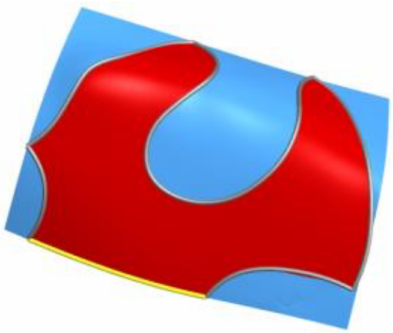
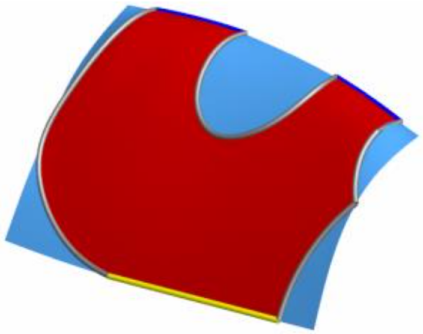
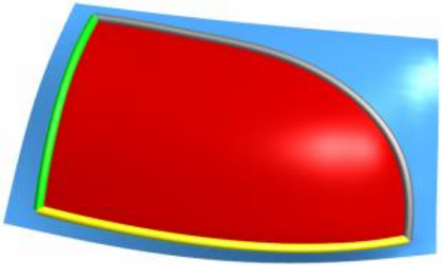
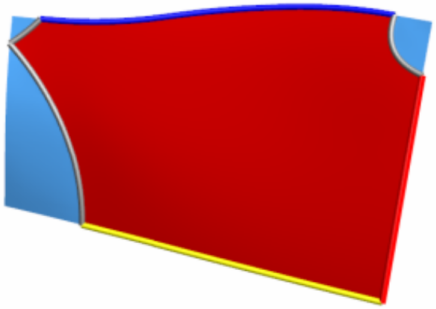
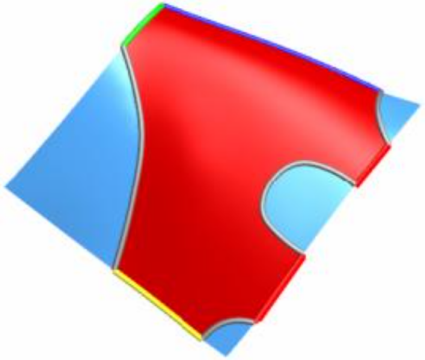
Admissible Label Configurations

		
a) $L = 0 / C = 0$	b) $L = 1 / C = 0$	c) $L = 2 / C = 0$
		
d) $L = 2 / C = 1$	e) $L = 3 / C = 2$	f) $L = 4 / C = 4$

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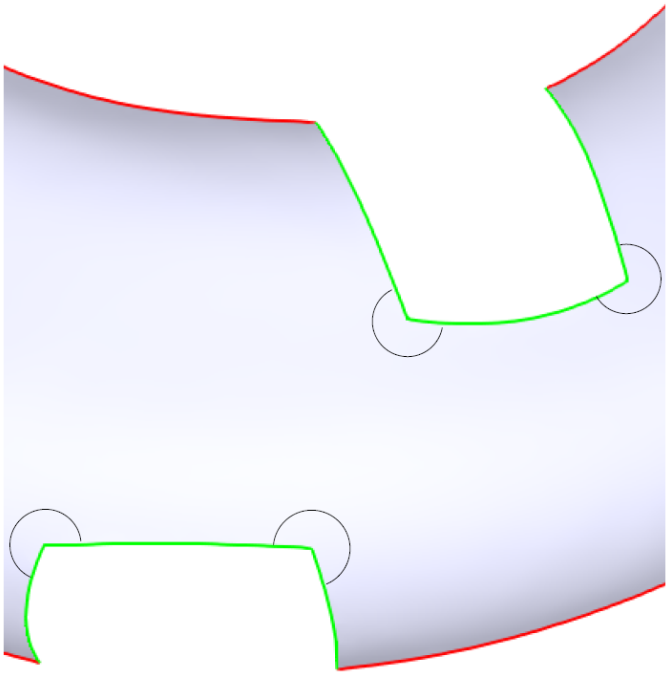
Algorithm for Automatic Labeling

- Input: mesh w/ segmented boundary loop
- Output: segment labels (North, West, South, East, Unlabeled)
- 1. Detect label candidates
- 2. Classify corner candidates
- 3. Concatenate label candidates through weak corners
- 4. while(label configuration not admissible)
  - 4.1. Remove weakest label candidate(s)
  - 4.2. Reclassify corner candidates
- 5. Assign labels of detected configuration

Detecting Label Candidates: Concave Corners

- Region has been created from rectangular patch by Boolean operations
- Segments with concave angles are trim curves

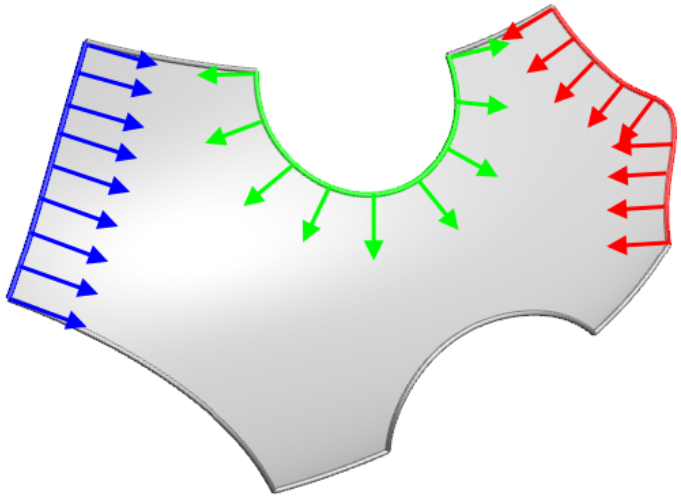


Detecting Label Candidates: Moving Frames

- Typically: "simple" isocurves, "complex" trimcurves
- Condition: moving frame rotation
- Measures geodesic curvature + corner angles

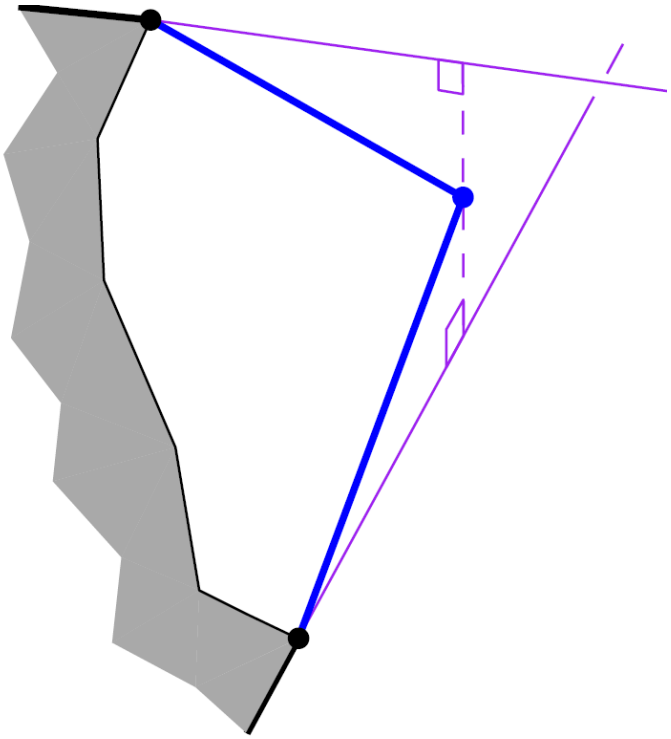
$$E_{\text{Label}} =\approx \int_L \kappa_g^2(s)ds$$

- Segments with large E_{Label} are trim curves



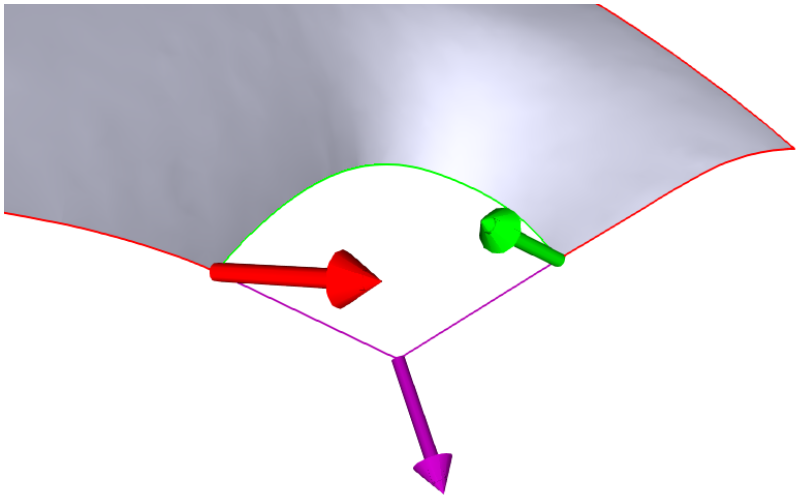
Classifying Corners

- Real and virtual corners
- Corners are to be classified:
  - Concave
  - Parallel
  - Weakly convex
  - Strongly convex
- Strong corners: close to 90 degrees, not far from the surface

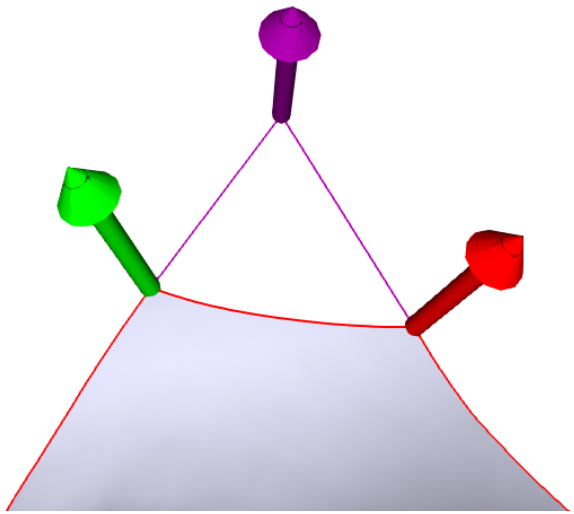


Classifying Corners: Trihedron Test

- Corners should bound regions of low curvature



Weak corner

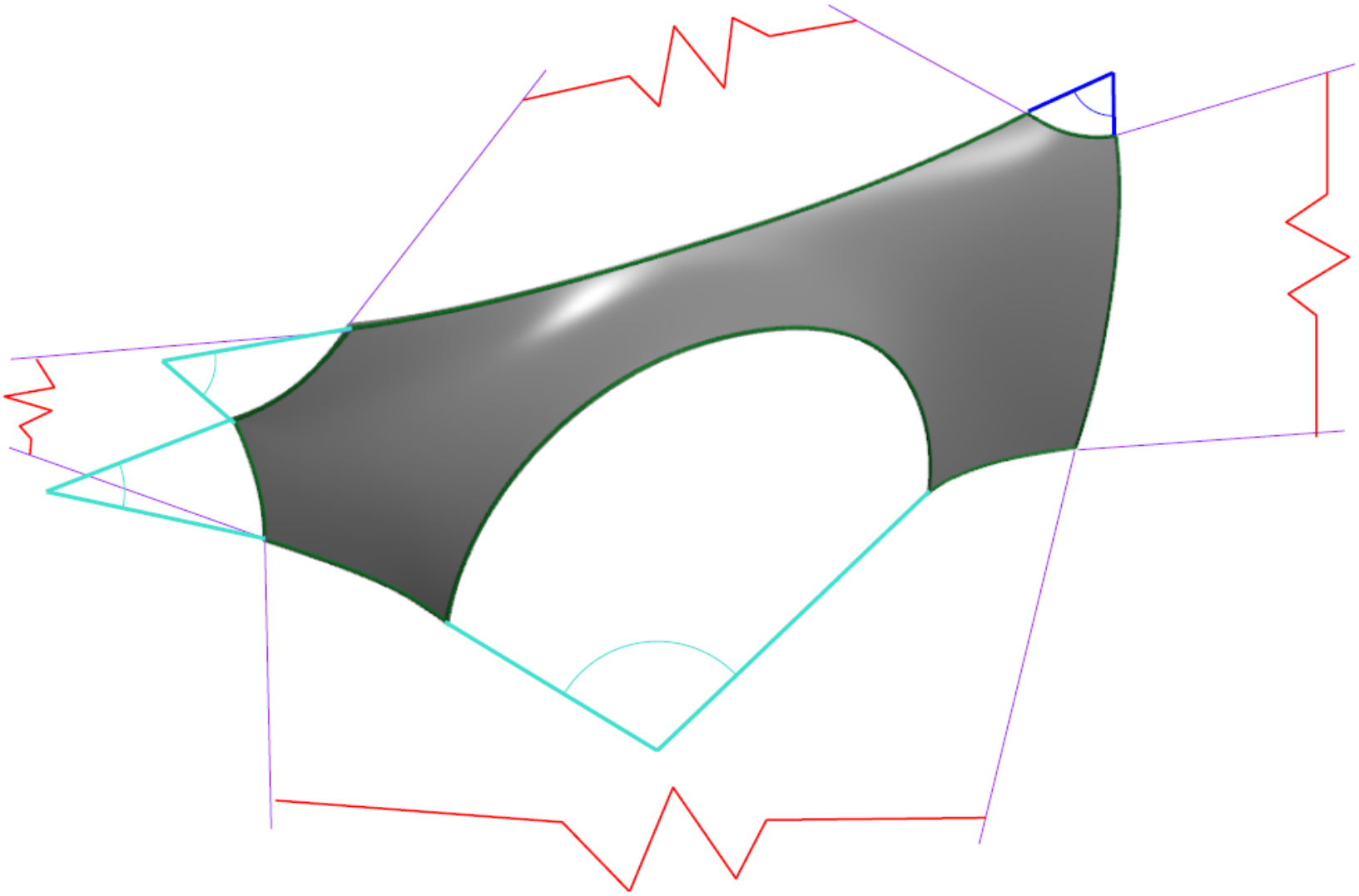


Strong corner

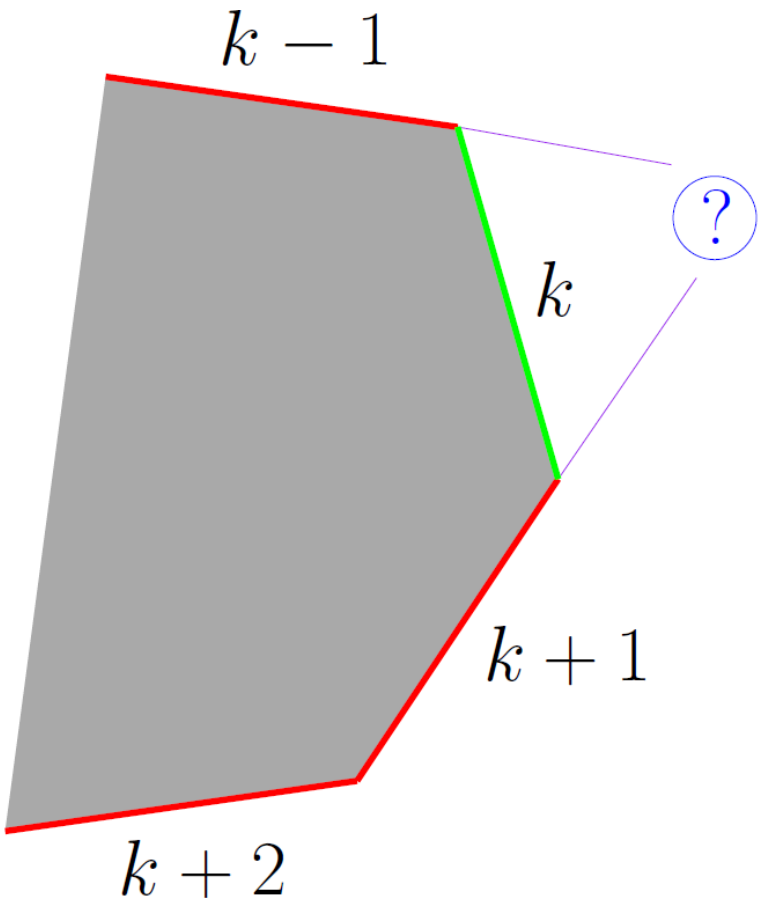
- Energy measuring corner strength:

$$E_{\text{Cor.}} = (1 - w)E_{\text{Cvx.}} + wE_{\text{Tri.}}$$

Classifying Corners: An Example

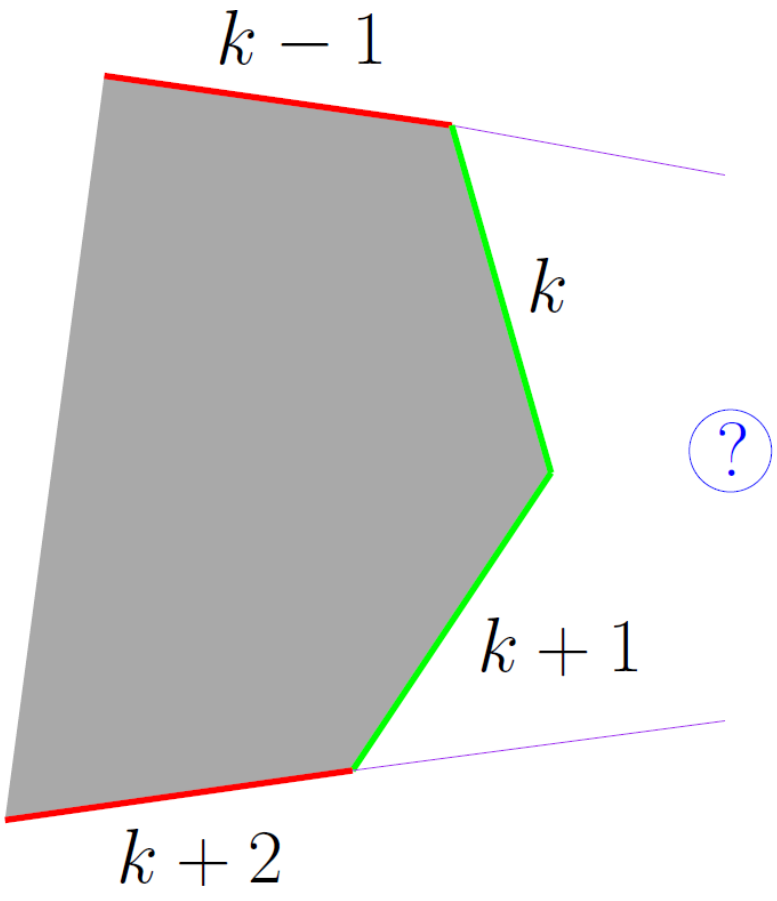


Removing Label Candidates



Label removal

$L \to L-1, C \to C-1$



Corner removal

$L \to L-2, C \to C-3$

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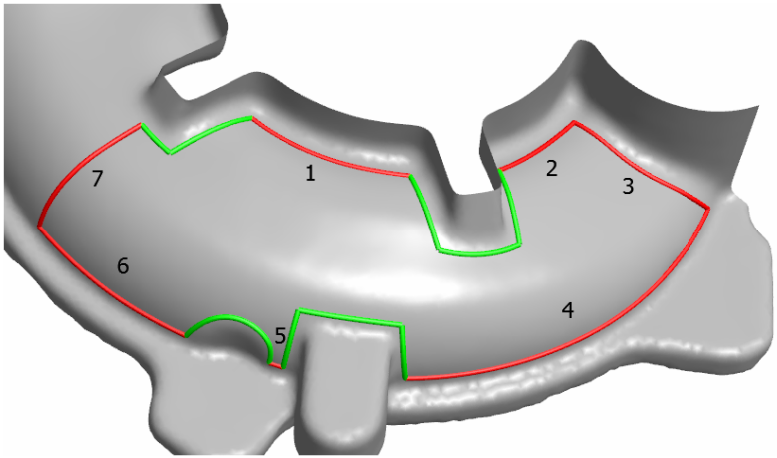
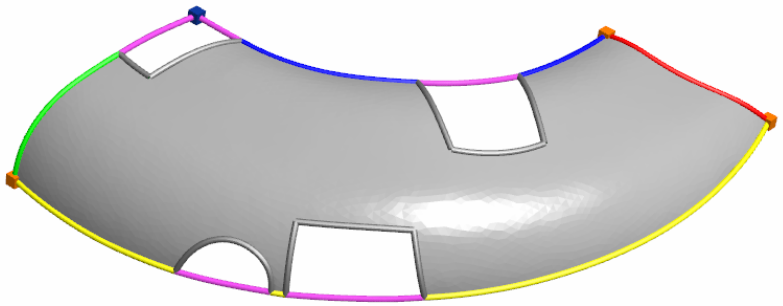
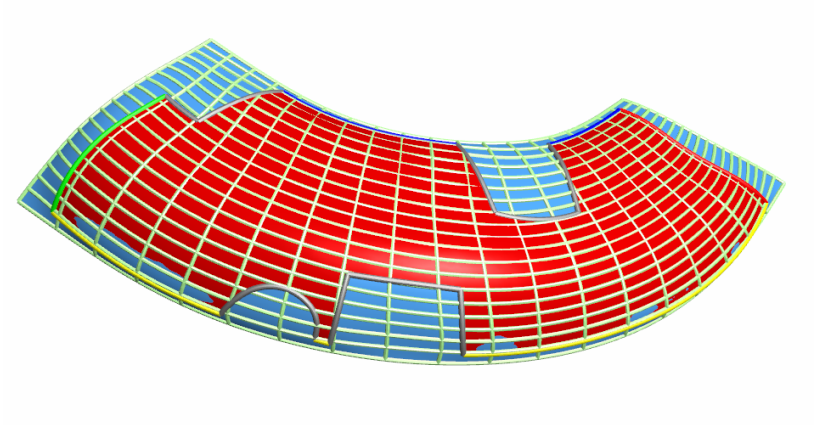
Labeling Results



Label Candidates

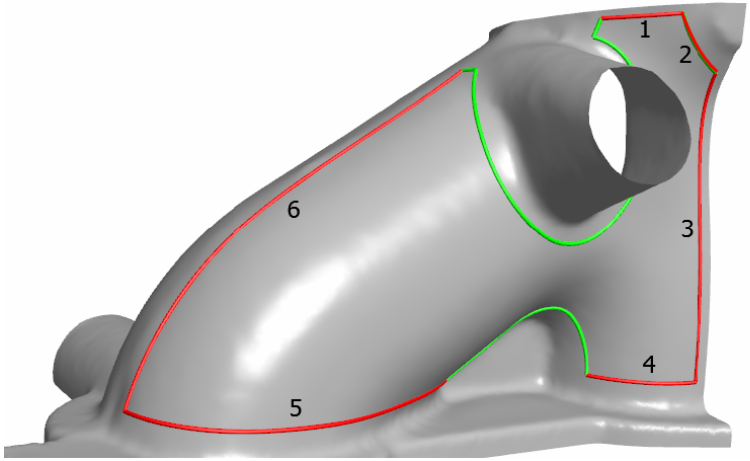


Corner Candidates

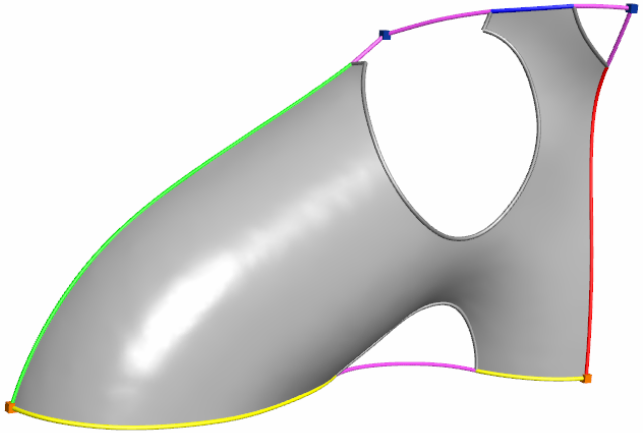


Detected Labeling: $L = 4, C = 4$

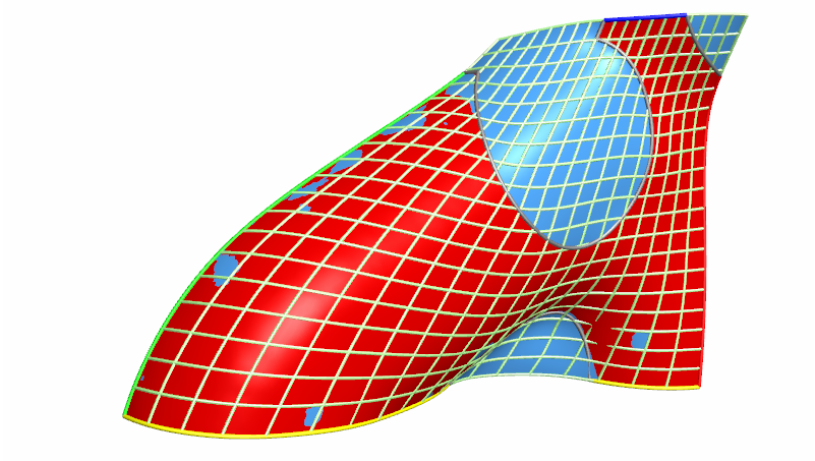
Labeling Results



Label Candidates

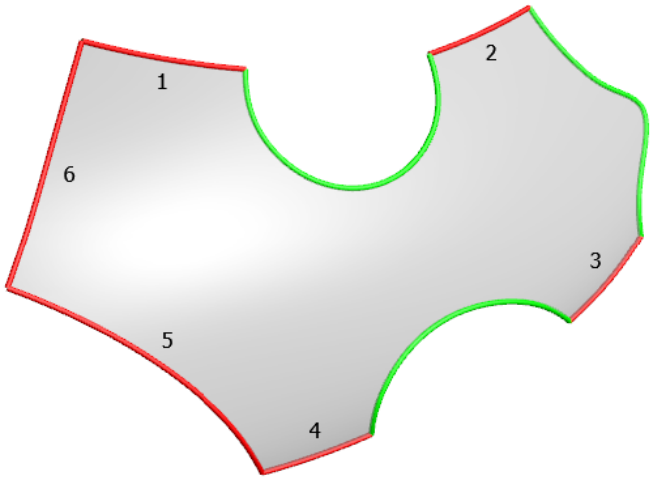


Corner Candidates

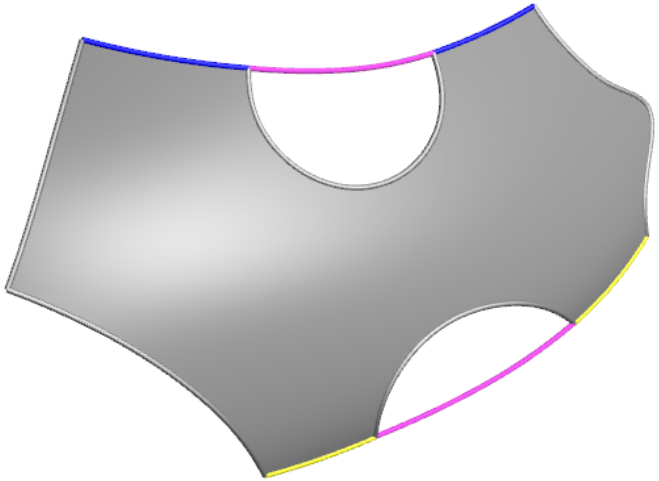


Detected Labeling: $L = 4, C = 4$

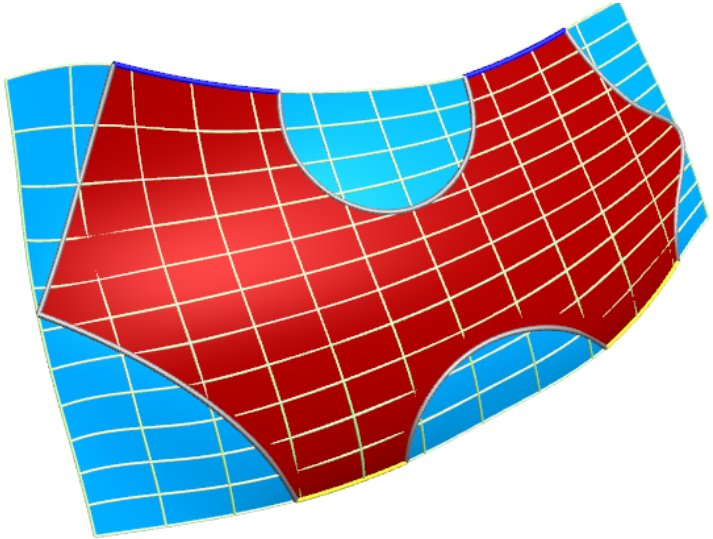
Labeling Results



Label Candidates

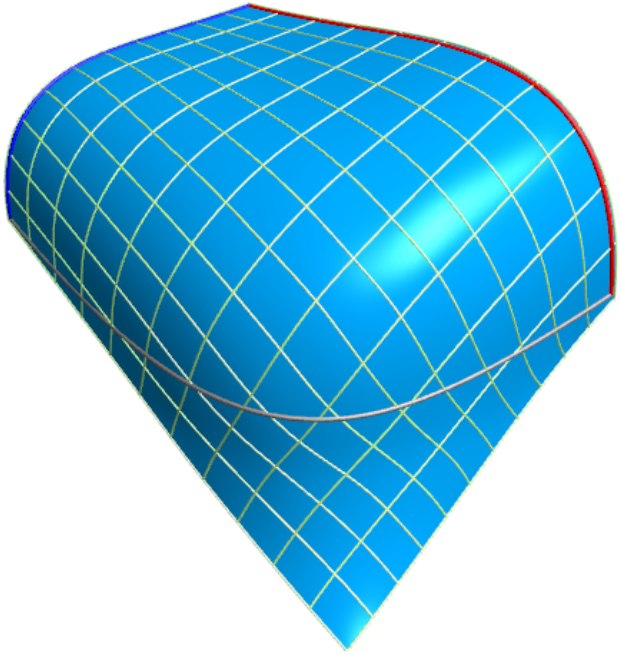


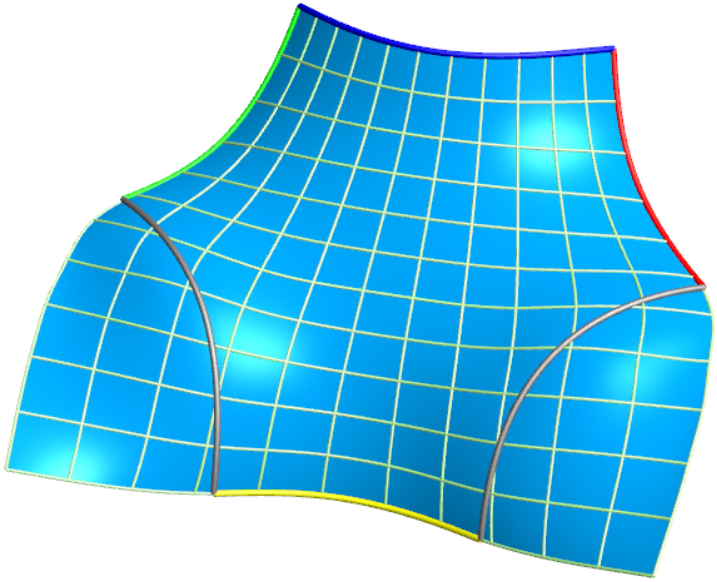
Corner Candidates



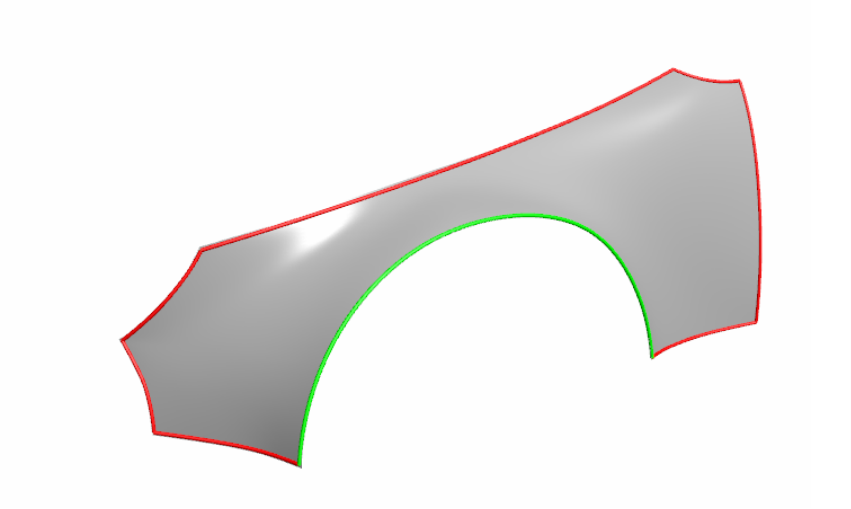
Detected Labeling: $L = 2, C = 0$

Limitations

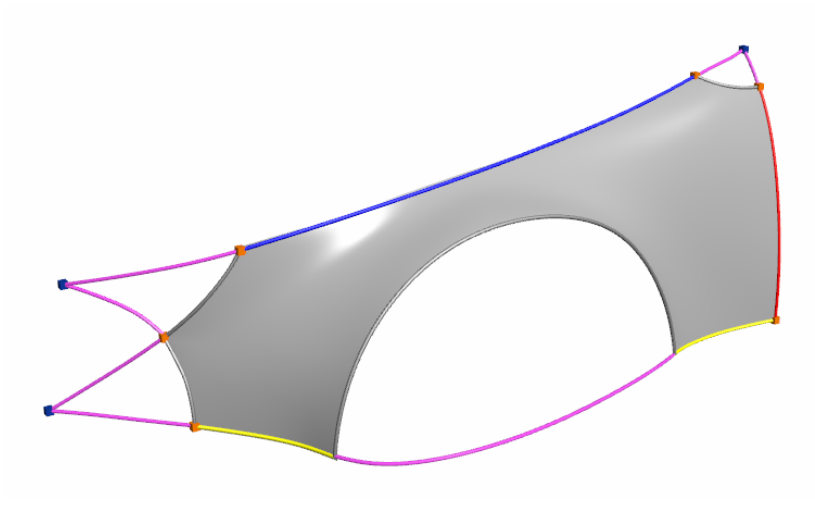




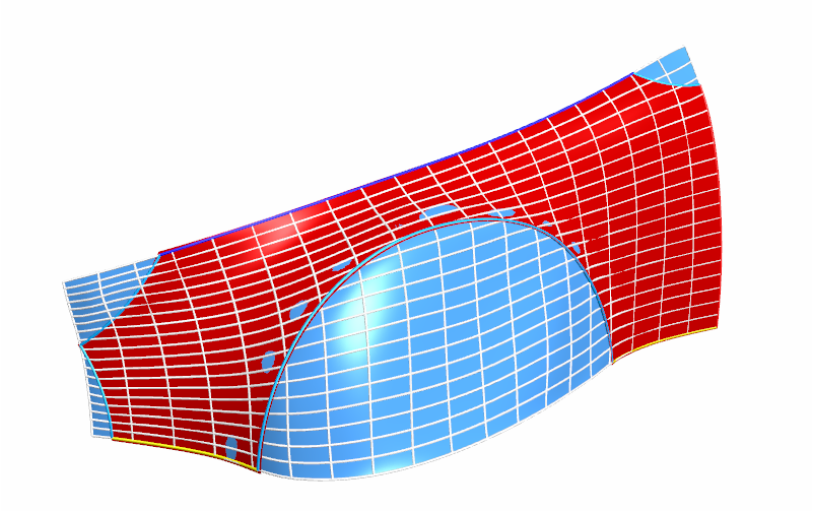
Ambiguous/highly symmetric cases require user intervention...



Label Candidates



Corner Candidates



Detected Labeling: $L = 3, C = 2$

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Conclusions

- Labeling: useful way to orient surfaces for tensor-product fitting
- Labeled parameterization
- Avoid weak control points by extension
- Algorithm for automatic labeling
- Future work:
  - Surface geometry (e.g. curvature alignment)?
  - Neighbouring regions?
  - Other uses of labeling?

Thank You for Your Attention!