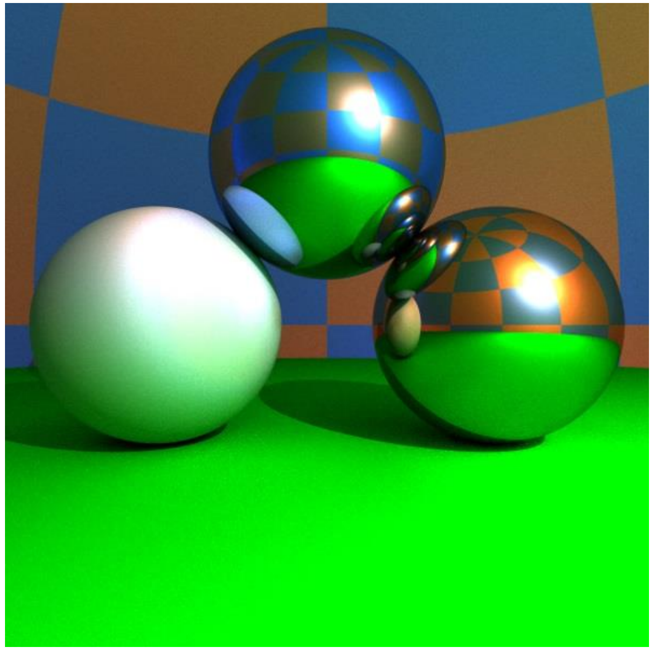
“As technology advances, the rendering time remains constant.”
Jim Blinn

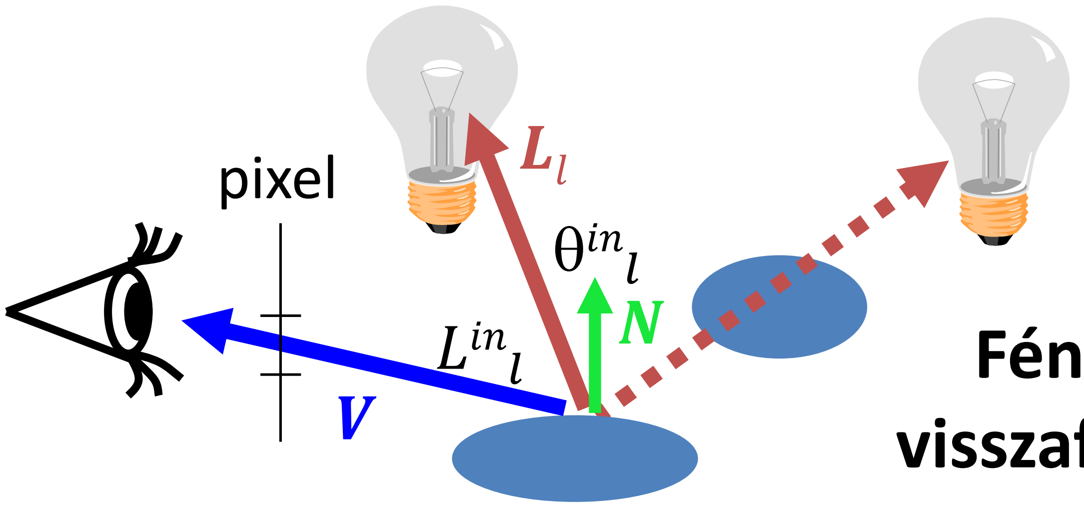
Sugárkövetés: ray-casting, ray-tracing, path-tracing

Szirmay-Kalos László



Lokális illumináció:
rücskös felületek, absztrakt fényforrások

Csak absztrakt
fényforrások
direkt megvilágítása



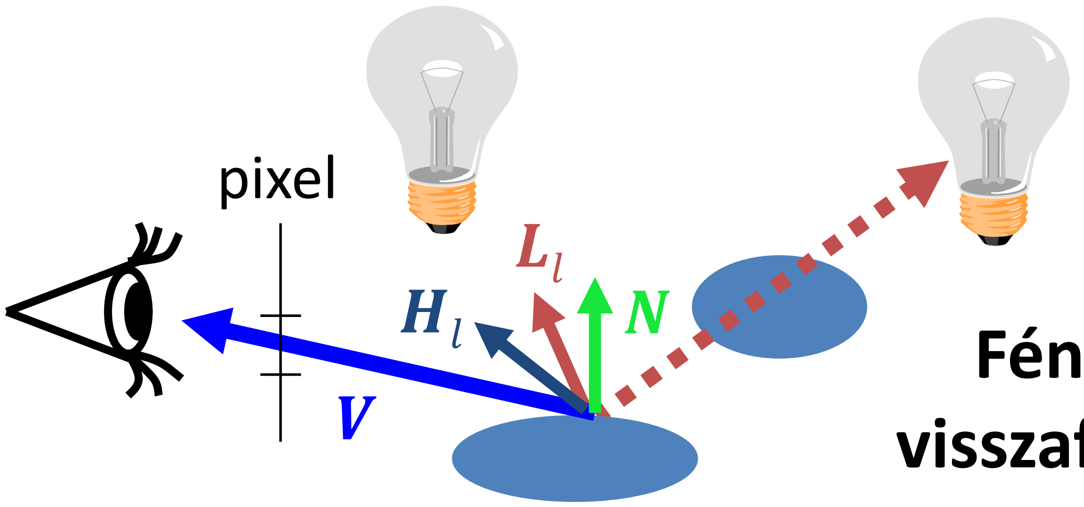
Fénysugarakat visszafelé követjük!

$$L\,(\boldsymbol{V}) \approx \sum_l L^{in}{}_l \; * f_r(\boldsymbol{L}_l, \boldsymbol{N}, \boldsymbol{V}) \cdot \cos^+ \theta^{in}{}_l$$

Absztrakt fényforrásokból származó megvilágítás.
(*Irányforrás = konstans; Pontforrás = távolság négyzetével csökken*
Ha takart, akkor zérus)

Lokális illumináció: rücskös felületek, absztrakt fényforrások

Csak absztrakt fényforrások direkt megvilágítása



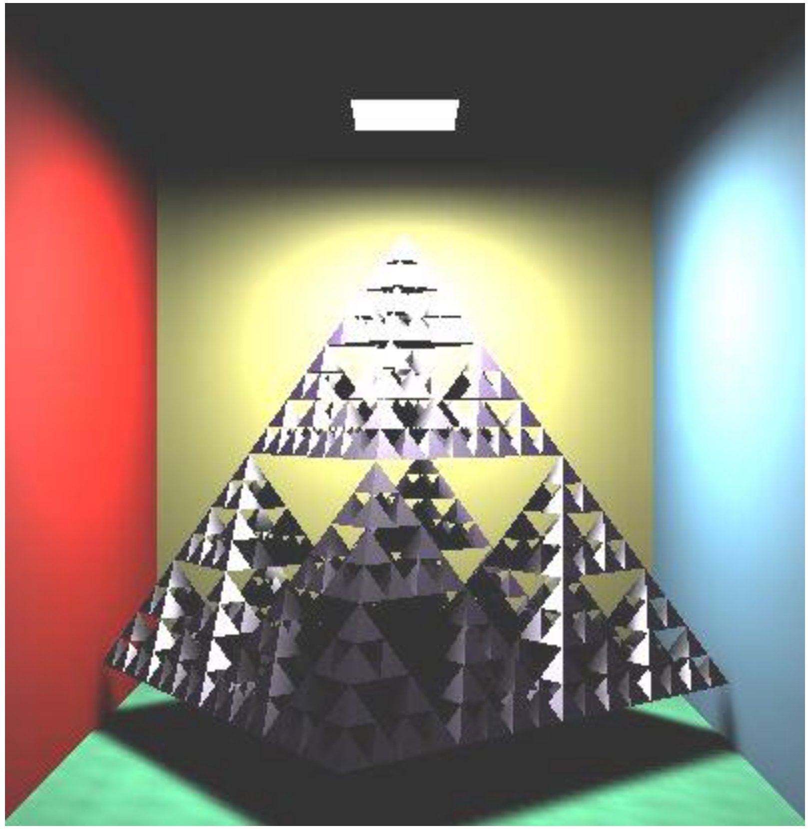
Fénysugarakat visszafelé követjük!

$$L(\boldsymbol{V}) \approx \sum_l L^{in}{}_l * \{k_d \cdot (\boldsymbol{L}_l \bullet \boldsymbol{N})^+ + k_s \cdot ((\boldsymbol{H}_l \bullet \boldsymbol{N})^+)^{shine}\}$$

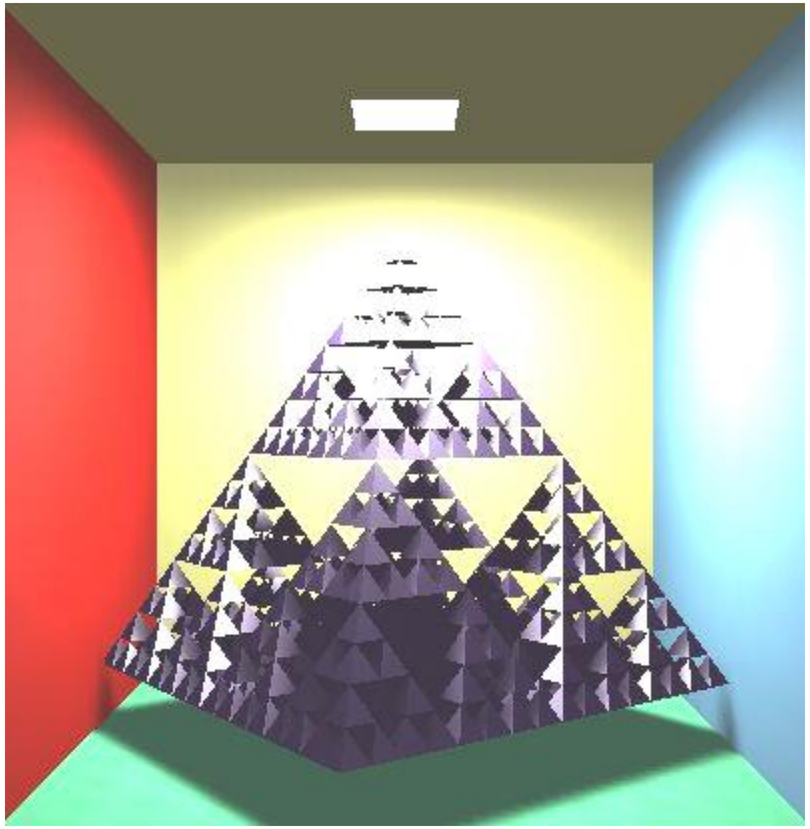
Absztrakt fényforrásokból származó megvilágítás.
(*Irányforrás = konstans; Pontforrás = távolság négyzetével csökken*
Ha takart, akkor zérus)

Ambiens tag

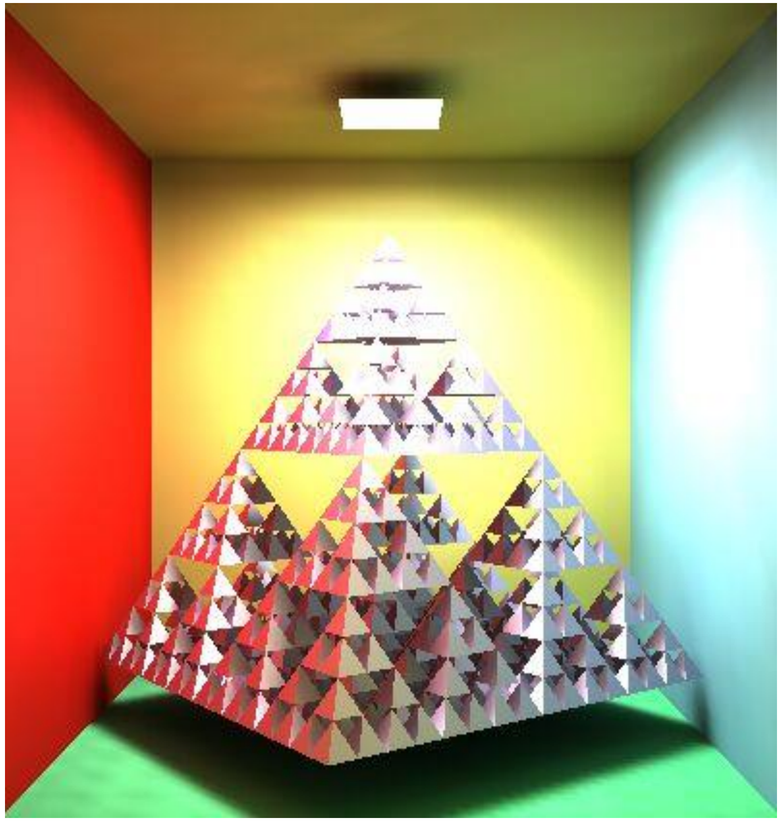
Lokális illumináció



+ ambiens tag

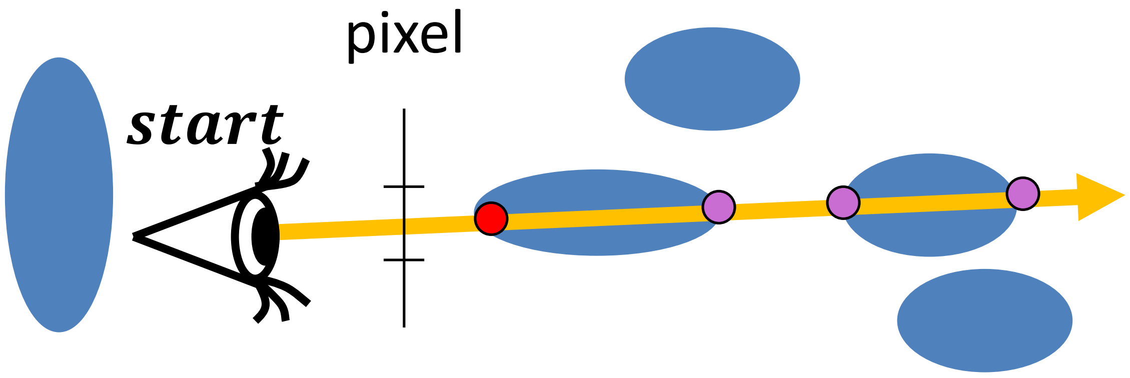


Globális illumináció



$$L\,(\boldsymbol{V}) \approx \sum_{l} L^{in}{}_{l} \;*\, f_r(\boldsymbol{L}_l, \boldsymbol{N}, \boldsymbol{V}) \cdot \cos^{+}\theta^{in}{}_{l} + \boxed{k_a * L_a}$$

Láthatóság



$$\boldsymbol{ray}(t) = \boldsymbol{start} + \boldsymbol{dir} \cdot t, \quad t > 0$$

```
struct Ray {
  vec3 start;
  vec3 dir;  // egységvektor
  bool out;  // kívül? vagy ior
};

struct Hit {
  float t;
  vec3 position;
  vec3 normal;
  Material* material;
  Hit() { t = -1; }
};
```

```
Hit firstIntersect(Ray ray) {
  Hit bestHit;
  for(Intersectable * obj : objects) {
    Hit hit = obj->intersect(ray); // hit.t < 0 ha nincs metszés
    if(hit.t > 0 && (bestHit.t < 0 || hit.t < bestHit.t)) bestHit = hit;
  }
  if (dot(ray.dir, bestHit.normal) > 0) bestHit.normal *= -1;
  return bestHit;
}
```

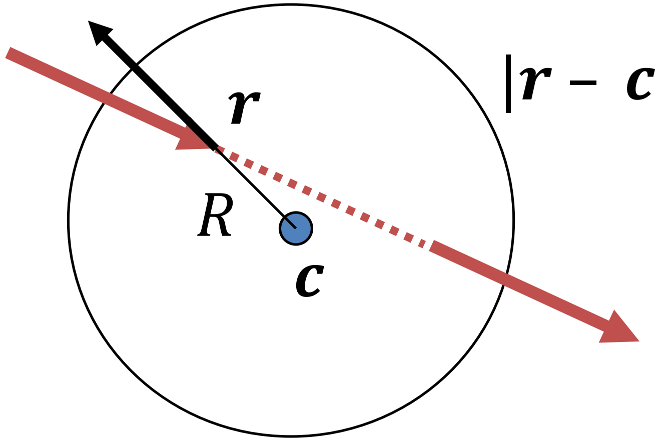
Nézzen felénk!

if $(\boldsymbol{N} \bullet \boldsymbol{dir} > 0)\ \{\ \boldsymbol{N} = -\boldsymbol{N};\ \}$

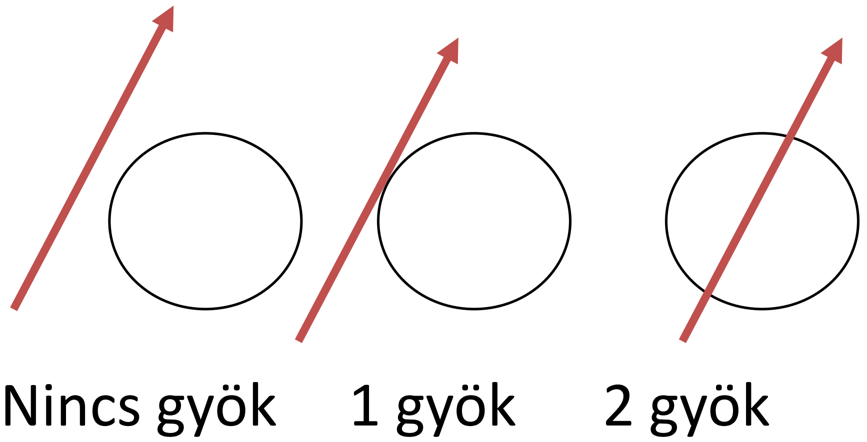
Objektum = Intersectable

```
struct Intersectable {
  Material* material;
  virtual Hit intersect(const Ray& ray) = 0;
};
```

Metszéspont számítás gömbbel



$|\boldsymbol{r} - \boldsymbol{c}|^2 = R^2$



$$\boldsymbol{ray}(t) = \boldsymbol{start} + \boldsymbol{dir} \cdot t$$

$$|\boldsymbol{ray}(t) - \boldsymbol{c}|^2 = (\boldsymbol{start} + \boldsymbol{dir} \cdot t - \boldsymbol{c}) \cdot (\boldsymbol{start} + \boldsymbol{dir} \cdot t - \boldsymbol{c}) = R^2$$

$$(\boldsymbol{dir} \cdot \boldsymbol{dir})t^2 + 2((\boldsymbol{start} - \boldsymbol{c}) \cdot \boldsymbol{dir})t + (\boldsymbol{start} - \boldsymbol{c}) \cdot (\boldsymbol{start} - \boldsymbol{c}) - R^2 = 0$$

Wanted: a pozitív megoldások közül a kisebb

Felületi normális gömbre: $\boldsymbol{N} = (\boldsymbol{ray}(t) - \boldsymbol{c})/R$

Sphere mint Intersectable

```
class Sphere : public Intersectable {
    vec3 center;
    float radius;
public:
    Hit intersect(const Ray& ray) {
        Hit hit;
        vec3 dist = ray.start - center;
        float a = dot(ray.dir, ray.dir);
        float b = dot(dist, ray.dir) * 2;
        float c = dot(dist, dist) - radius * radius;
        float discr = b * b - 4 * a * c;
        if (discr < 0) return hit; else discr = sqrtf(discr);
        float t1 = (-b + discr)/2/a, t2 = (-b - discr)/2/a;
        if (t1 <= 0) return hit; // t1 >= t2 for sure
        hit.t = (t2 > 0) ? t2 : t1;
        hit.position = ray.start + ray.dir * hit.t;
        hit.normal = (hit.position - center)/radius; // Only for sphere!!!
        hit.material = material;
        return hit;
    }
};
```

Implicit felületek

- A felület pontjai: $f(\boldsymbol{r}) = 0$
- Sugár: $\boldsymbol{ray}(t) = \boldsymbol{s} + \boldsymbol{d} \cdot t$
- Metszés paraméter t^*: $f(\boldsymbol{s} + \boldsymbol{d} \cdot t) = 0$
- Metszéspont: $\boldsymbol{r}^* = \boldsymbol{ray}(t^*) = \boldsymbol{s} + \boldsymbol{d} \cdot t^*$
- Normálvektor $= \nabla f(\boldsymbol{r}^*)$

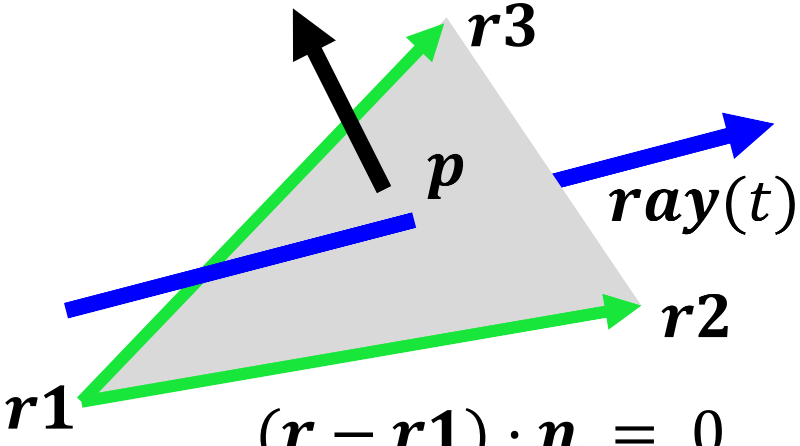
- **Kvadratikus felületek**: $f(\boldsymbol{r}) = [\boldsymbol{r}, 1] \cdot \boldsymbol{Q} \cdot [\boldsymbol{r}, 1]^T = 0$, ($\boldsymbol{Q}$ szimmetrikus)
- Metszés paraméter:

  $$[\boldsymbol{s} + \boldsymbol{d} \cdot t, 1] \cdot \boldsymbol{Q} \cdot [\boldsymbol{s} + \boldsymbol{d} \cdot t, 1]^T = 0$$

  $$[\boldsymbol{d}, 0] \cdot \boldsymbol{Q} \cdot [\boldsymbol{d}, 0]^T t^2 + 2[\boldsymbol{s}, 1] \cdot \boldsymbol{Q} \cdot [\boldsymbol{d}, 0]^T t + [\boldsymbol{s}, 1] \cdot \boldsymbol{Q} \cdot [\boldsymbol{s}, 1]^T = 0$$
- Normálvektor: $\nabla f(\boldsymbol{r}^*) = \boldsymbol{Q} \cdot [\boldsymbol{r}^*, 1]^T$ első három koord.

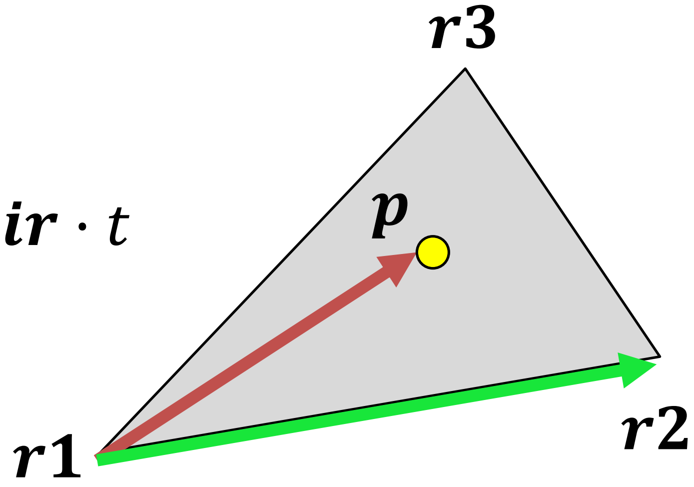
Háromszög (konvex sokszög)

$$\boldsymbol{n} = (\boldsymbol{r2} - \boldsymbol{r1}) \times (\boldsymbol{r3} - \boldsymbol{r1})$$



$$\boldsymbol{ray}(t) = \boldsymbol{start} + \boldsymbol{dir} \cdot t$$

$$(\boldsymbol{r} - \boldsymbol{r1}) \cdot \boldsymbol{n} = 0$$



1. Síkmetszés: $(\boldsymbol{ray}(t) - \boldsymbol{r1}) \cdot \boldsymbol{n} = 0, \quad t > 0$

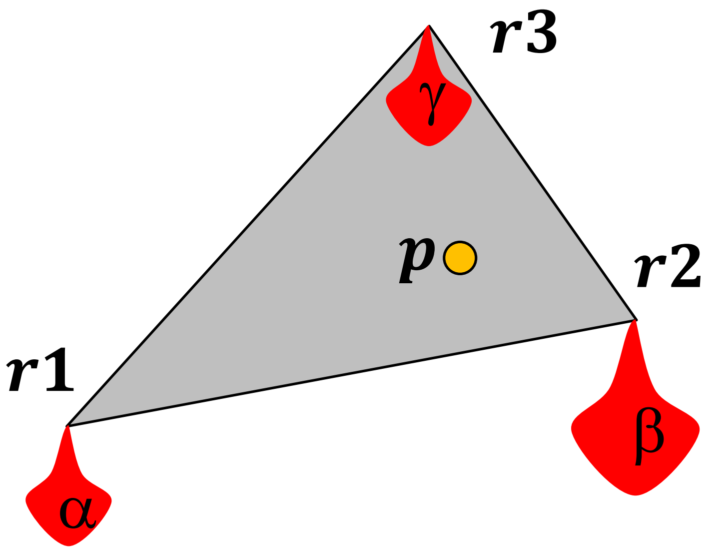
$$t = \frac{(\boldsymbol{r1} - \boldsymbol{start}) \cdot \boldsymbol{n}}{\boldsymbol{dir} \cdot \boldsymbol{n}}$$

2. A metszéspont a háromszögön belül van-e?

$$((\boldsymbol{r2} - \boldsymbol{r1}) \times (\boldsymbol{p} - \boldsymbol{r1})) \cdot \boldsymbol{n} > 0$$
$$((\boldsymbol{r3} - \boldsymbol{r2}) \times (\boldsymbol{p} - \boldsymbol{r2})) \cdot \boldsymbol{n} > 0$$
$$((\boldsymbol{r1} - \boldsymbol{r3}) \times (\boldsymbol{p} - \boldsymbol{r3})) \cdot \boldsymbol{n} > 0$$

Felületi normális: $\boldsymbol{n}$
vagy árnyaló normálok
(shading normals)

Háromszög metszés baricentrikus koordinátákban



$$\boldsymbol{p}(\alpha,\beta,\gamma) = \alpha\cdot\boldsymbol{r1} + \beta\cdot\boldsymbol{r2} + \gamma\cdot\boldsymbol{r3}$$

$\alpha+\beta+\gamma = 1$ 1 skalár egyenlet

$\alpha,\beta,\gamma \geq 0$

$$\boldsymbol{ray}(t) = \boldsymbol{p}(\alpha,\beta,\gamma)$$

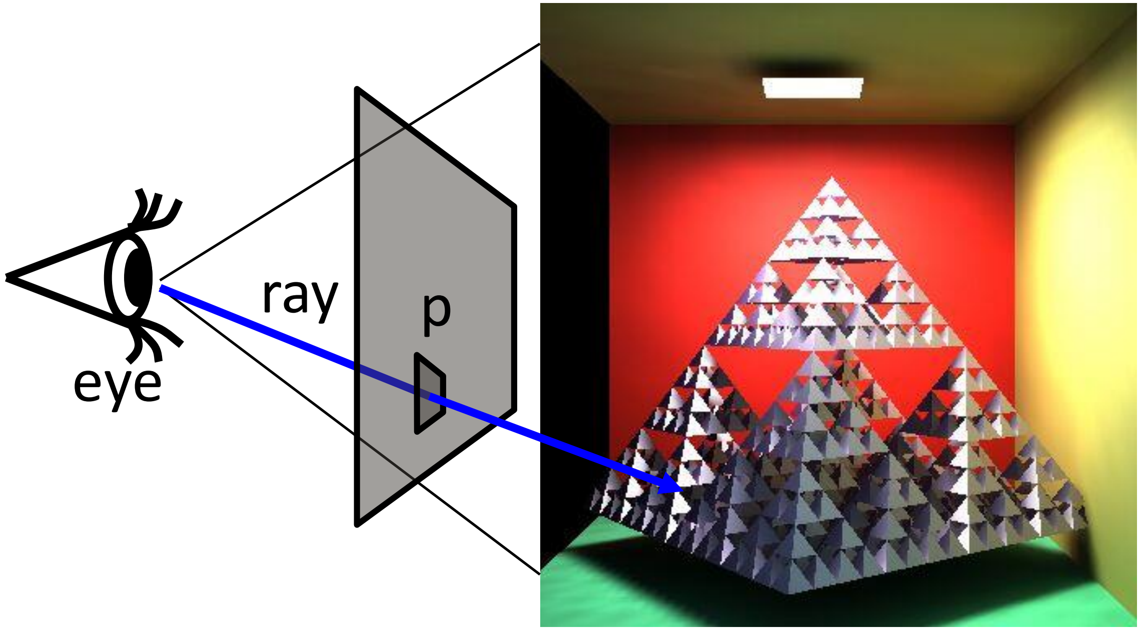
$\boldsymbol{start} + \boldsymbol{dir}\cdot t = \alpha\cdot\boldsymbol{r1} + \beta\cdot\boldsymbol{r2} + \gamma\cdot\boldsymbol{r3}$ 3 skalár egyenlet

4 ismeretlen: α,β,γ,t

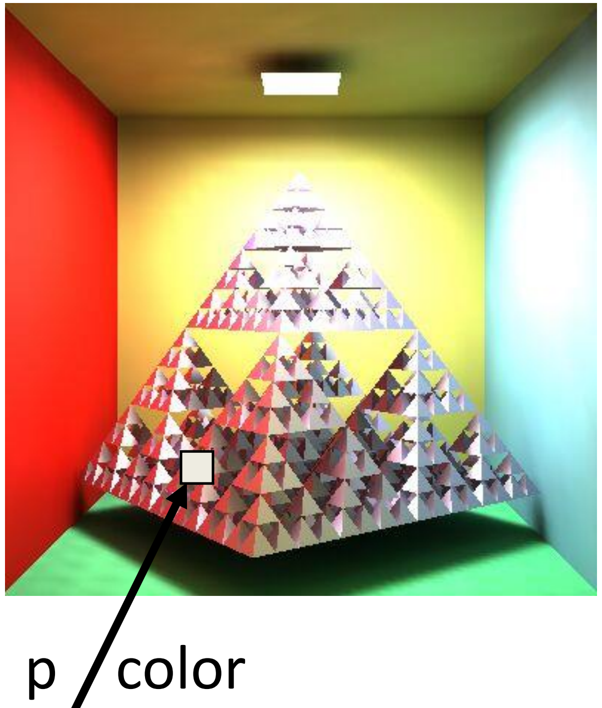
A megoldás után ellenőrzés, hogy mind nem negatív-e

Sugárkövetés: Render

Virtuális világ: szem+ablak



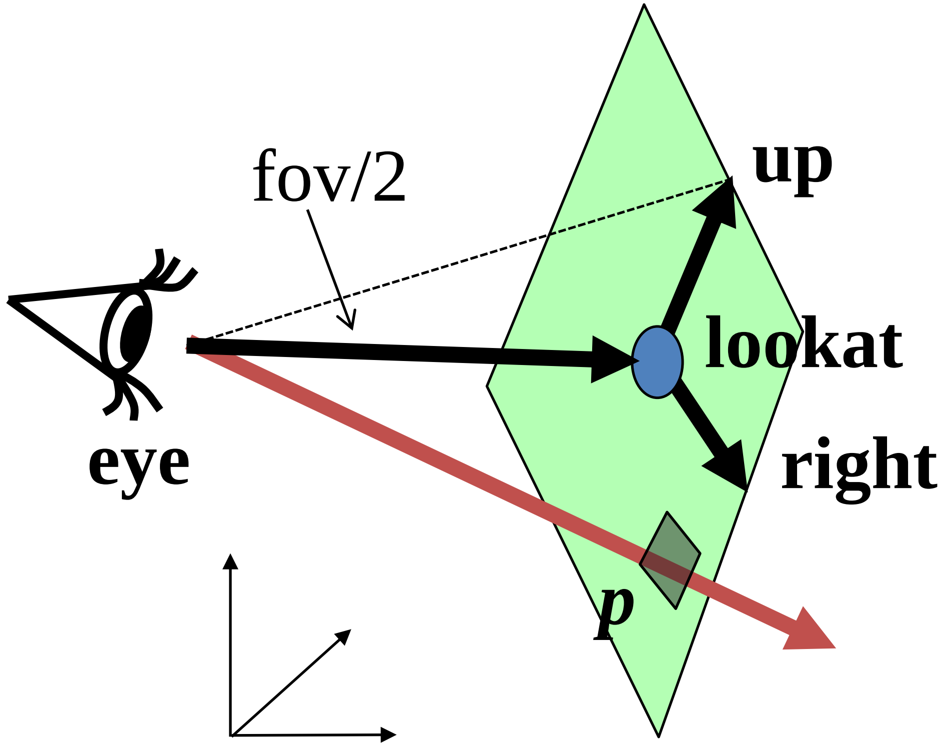
Valós világ: néző+képernyő

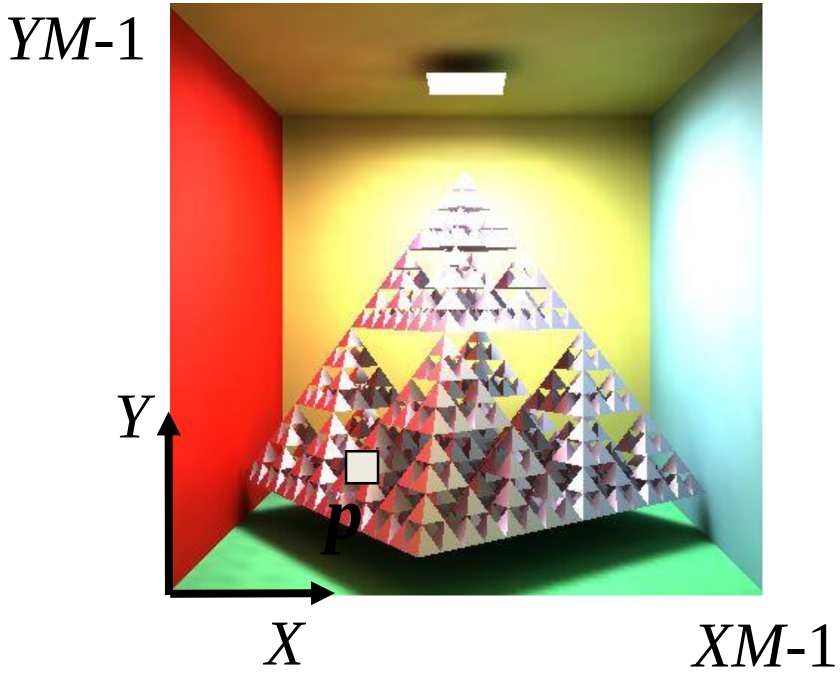


<u>Render( )</u>

```
for each pixel p
        Ray r = getRay( eye → pixel p )
        color = trace(ray)
        WritePixel(p, color)
endfor
end
```

Kamera: getRay



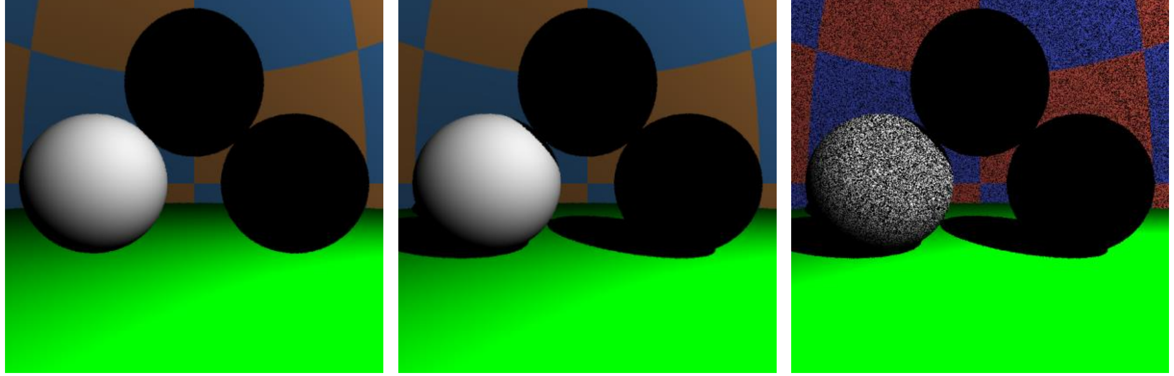


Normalizált eszköz koordináták

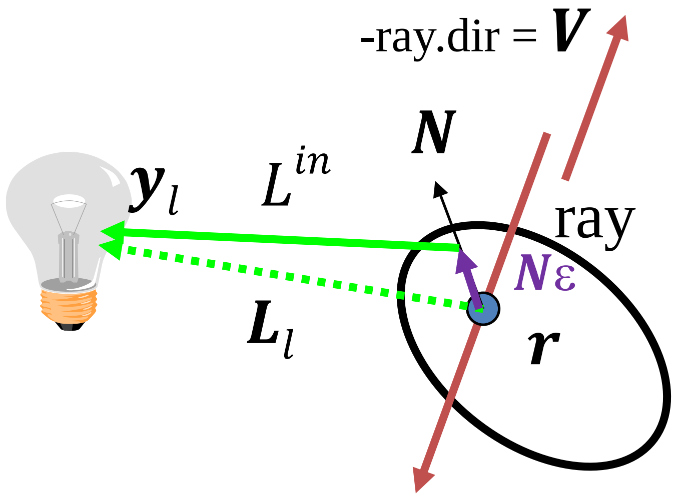
$$\boldsymbol{p} = \mathbf{lookat} + \alpha \cdot \mathbf{right} + \beta \cdot \mathbf{up}, \qquad \alpha, \beta \text{ in } [-1,1]$$

$$= \mathbf{lookat} + (2(X+0.5)/XM-1) \cdot \mathbf{right} + (2(Y+0.5)/YM-1) \cdot \mathbf{up}$$

Ray: start = eye, dir =p – eye



No Shadow | Shadow | $\varepsilon = 0$: acne



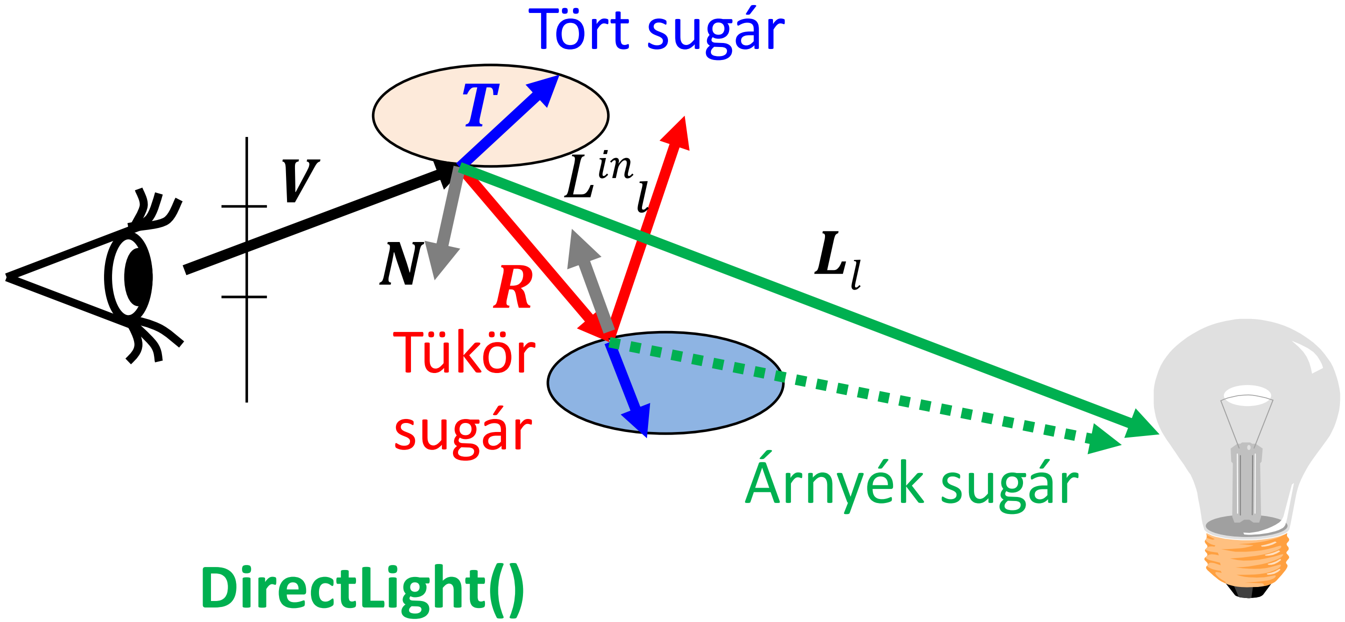

```
vec3 trace(Ray ray) {
    Hit hit = firstIntersect(ray);
    if(hit.t < 0) return L_a; // nothing
    [r, N, k_a, k_d, k_s, shine] ← hit;
    vec3 outRad = k_a * L_a;
    for(each light source l){
        Ray shadowRay(r + Nε, L_l);
        Hit shadowHit = firstIntersect(shadowRay);
        if(shadowHit.t < 0 || shadowHit.t > |r − y_l|)
            outRad += L^in_l * {k_d·(L_l•N)^+ + k_s·((H_l•N)^+)^shine}
    }
    return outRad;
}
```

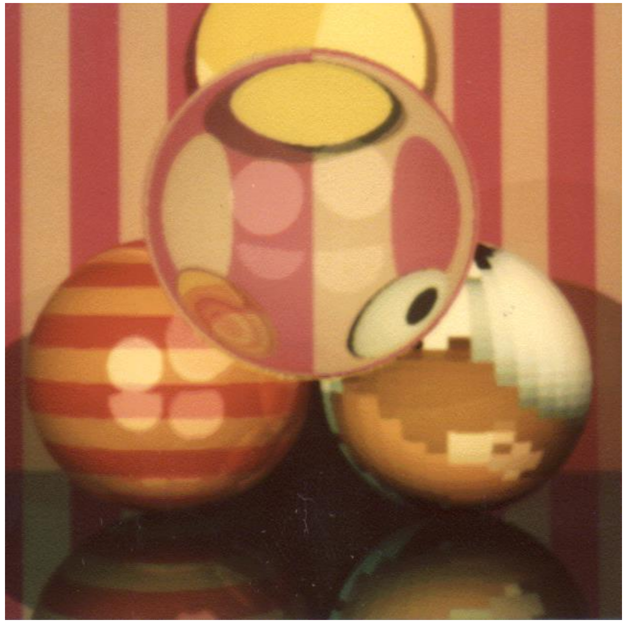
DirectLight ()

Shadow

$$L^{in}_l * \{k_d \cdot (\boldsymbol{L}_l \bullet \boldsymbol{N})^+ + k_s \cdot ((\boldsymbol{H}_l \bullet \boldsymbol{N})^+)^{shine}\}$$

Rekurzív sugárkövetés





DirectLight()

$$L\,(\boldsymbol{V}) \approx \begin{cases} k_a * L_a + \sum_l L^{in}{}_l \; * \{k_d \cdot (\boldsymbol{L}_l \bullet \boldsymbol{N})^+ + k_s \cdot ((\boldsymbol{H}_l \bullet \boldsymbol{N})^+)^{shine}\} \\ \boxed{F(\boldsymbol{V} \bullet \boldsymbol{N}) * L^{in}(\boldsymbol{R}) + (1 - F(\boldsymbol{V} \bullet \boldsymbol{N})) * L^{in}(\boldsymbol{T})} \end{cases}$$

Fresnel

Tükör irányból érkező fény

1-Fresnel

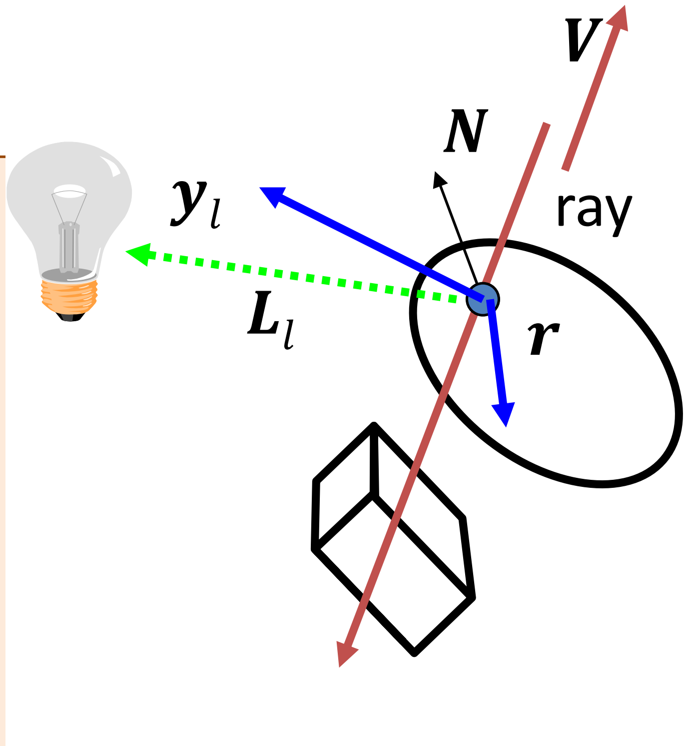
Törési irányból érkező fény

trace

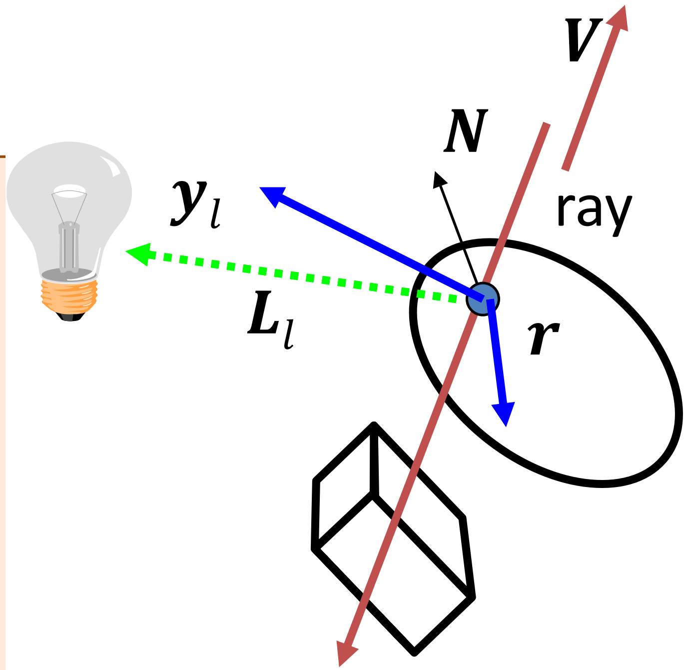
```
vec3 trace(Ray ray) {

  Hit hit = firstIntersect(ray); // [r,N,n,κ,k_a,k_d,k_s,shine] ← hit;
  if(hit.t < 0) return L_a; // nothing
  vec3 outRad(0, 0, 0);
  if(hit.material->rough) outRad = DirectLight(hit);

  if(hit.material->reflective){
     Ray reflectRay(r + Nε, reflect(ray.dir, N), ray.out);
     outRad += trace(reflectRay) * Fresnel(ray.dir, N);
  }
  if(hit.material->refractive) {
     ior = (ray.out) ? n.x : 1/n.x;
     vec3 refractionDir = refract(ray.dir,N,ior);
     if (length(refractionDir) > 0) {
        Ray refractRay(r - Nε, refractionDir, !ray.out);
        outRad += trace(refractRay)*(vec3(1,1,1) - Fresnel(ray.dir, N));
     }
  }
  return outRad;
}
```



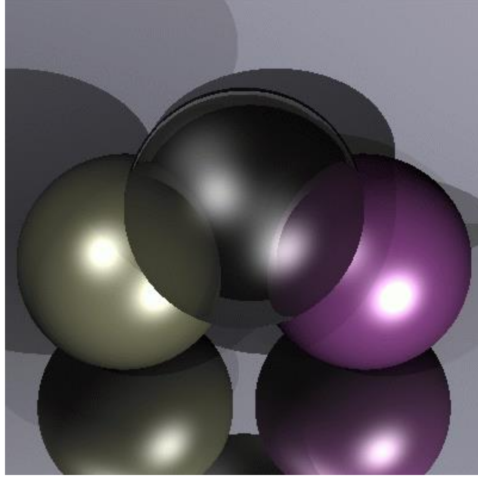
trace




```
vec3 trace(Ray ray, int d=0) {
  if (d > maxdepth) return La;
  Hit hit = firstIntersect(ray); //[r,N,n,κ,ka,kd,ks,shine] ← hit;
  if(hit.t < 0) return La; // nothing
  vec3 outRad(0, 0, 0);
  if(hit.material->rough) outRad = DirectLight(hit);

  if(hit.material->reflective){
     Ray reflectRay(r + Nε, reflect(ray.dir, N), ray.out);
     outRad += trace(reflectRay, d+1) * Fresnel(ray.dir, N)
  }
  if(hit.material->refractive) {
     ior = (ray.out) ? n.x : 1/n.x;
     vec3 refractionDir = refract(ray.dir,N,ior);
     if (length(refractionDir) > 0) {
        Ray refractRay(r - Nε, refractionDir, !ray.out);
        outRad += trace(refractRay, d+1) * (vec3(1,1,1)-Fresnel(ray.dir, N));
     }
  }
  return outRad;
}
```

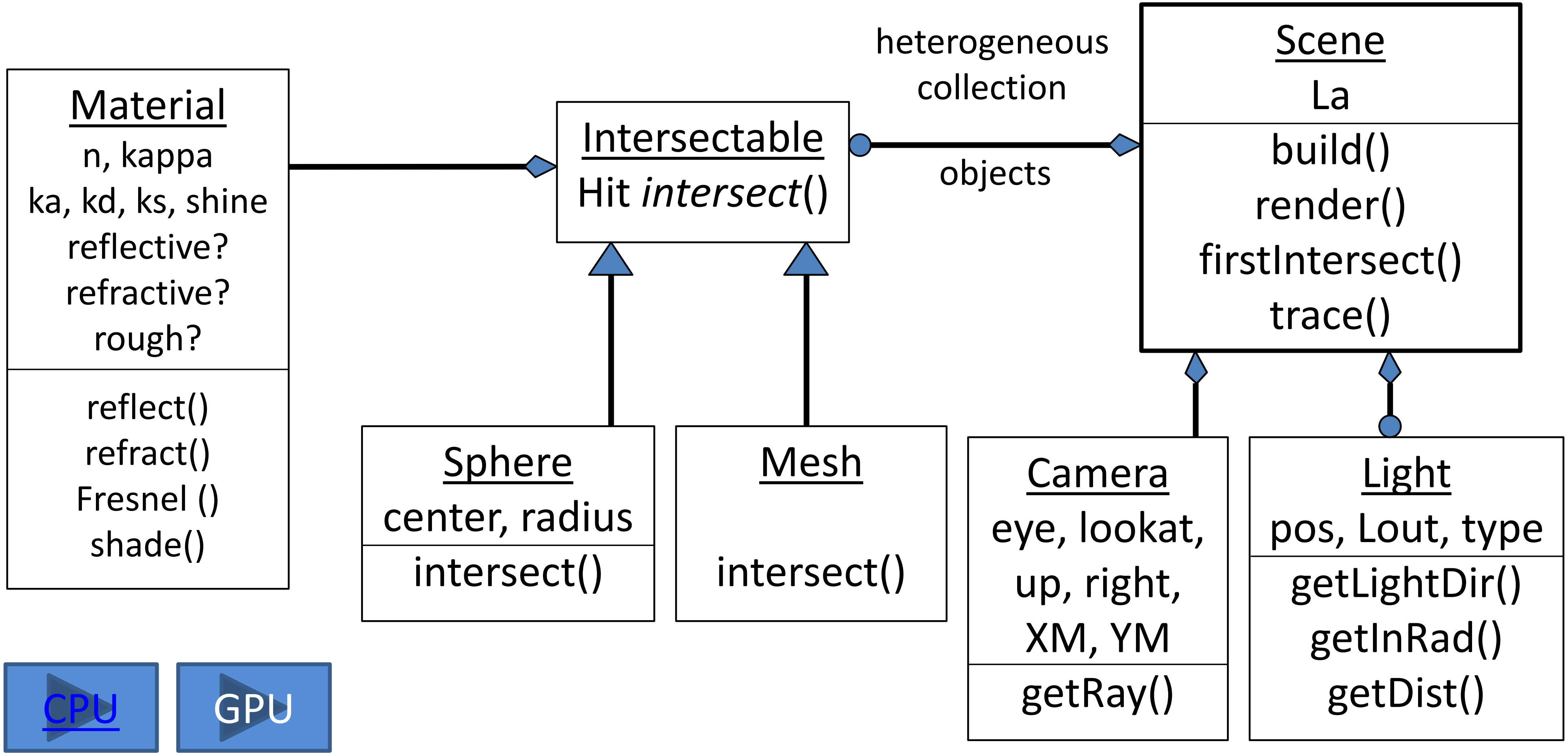
Paul Heckbert névjegye



```
typedef struct{double x,y,z}vec;vec U,black,amb={.02,.02,.02}; struct sphere{ vec cen,color;double rad,kd,ks,kt,kl,ir}*s,
*best,sph[]={0.,6.,.5,1.,1.,1.,.9, .05,.2,.85,0.,1.7,-1.,8.,-.5,1.,.5,.2,1.,.7,.3,0.,.05,1.2,1.,8.,-.5,.1,.8,.8, 1.,.3,.7,0.,0.,1.2,3.,-6.,15.,1.,
.8,1.,7.,0.,0.,0.,.6,1.5,-3.,-3.,12.,.8,1., 1.,5.,0.,0.,0.,.5,1.5,};yx; double u,b,tmin,sqrt(),tan();double vdot(A,B)vec A ,B;
{return A.x*B.x+A.y*B.y+A.z*B.z;}vec vcomb(a,A,B)double a;vec A,B; {B.x+=a* A.x;B.y+=a*A.y;B.z+=a*A.z;return B;}
vec vunit(A)vec A;{return vcomb(1./sqrt( vdot(A,A)),A,black);}struct sphere *intersect(P,D)vec P,D;{best=0;tmin=1e30;
s= sph+5;while(s-->sph)b=vdot(D,U=vcomb(-1.,P,s->cen)),u=b*b-vdot(U,U)+s->rad*s ->rad,u=u>0?sqrt(u):1e31,u=b-u>
1e-7?b-u:b+u,tmin=u>=1e-7&&u<tmin?best=s,u: tmin;return best;}vec trace(level,P,D)vec P,D;{double d,eta,e;vec N,color;
struct sphere*s,*l;if(!level--)return black;if(s=intersect(P,D));else return amb;color=amb;eta=s->ir;d= -vdot(D,N=vunit(vcomb
(-1.,P=vcomb(tmin,D,P),s->cen )));if(d<0)N=vcomb(-1.,N,black),eta=1/eta,d= -d;l=sph+5;while(l-->sph)if((e=l ->kl*vdot(N,
U=vunit(vcomb(-1.,P,l->cen))))>0&&intersect(P,U)==l)color=vcomb(e ,l->color,color);U=s->color;color.x*=U.x;color.y*=
U.y;color.z*=U.z;e=1-eta* eta*(1-d*d);return vcomb(s->kt,e>0?trace(level,P,vcomb(eta,D,vcomb(eta*d-sqrt (e),N,black))):
black,vcomb(s->ks,trace(level,P,vcomb(2*d,N,D)),vcomb(s->kd, color,vcomb(s->kl,U,black))));}

main(){printf("%d %d\n",32,32);while(yx<32*32) U.x=yx%32-32/2,U.z=32/2-yx++/32,U.y=32/2/tan(25/114.5915590261),
U=vcomb(255., trace(3,black,vunit(U)),black),printf("%.0f %.0f %.0f\n",U);}/*minray!*/
```

OO dekompozíció



CPU GPU

"Bad programmers worry about the code. Good programmes worry about data structures."

Linus Torvalds

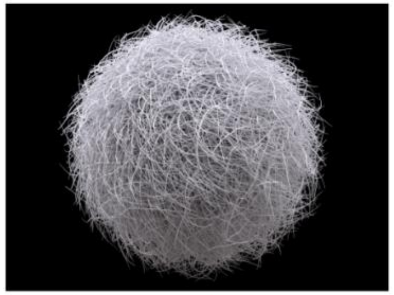
Térpartícionáló adatstruktúrák

Szirmay-Kalos László

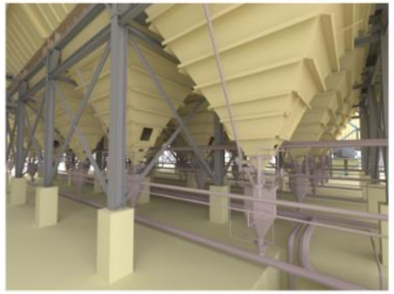
(a) CONFERENCE (282K triangles)

(b) CRYTEK SPONZA (262K triangles)

(c) FAIRY (174K triangles)



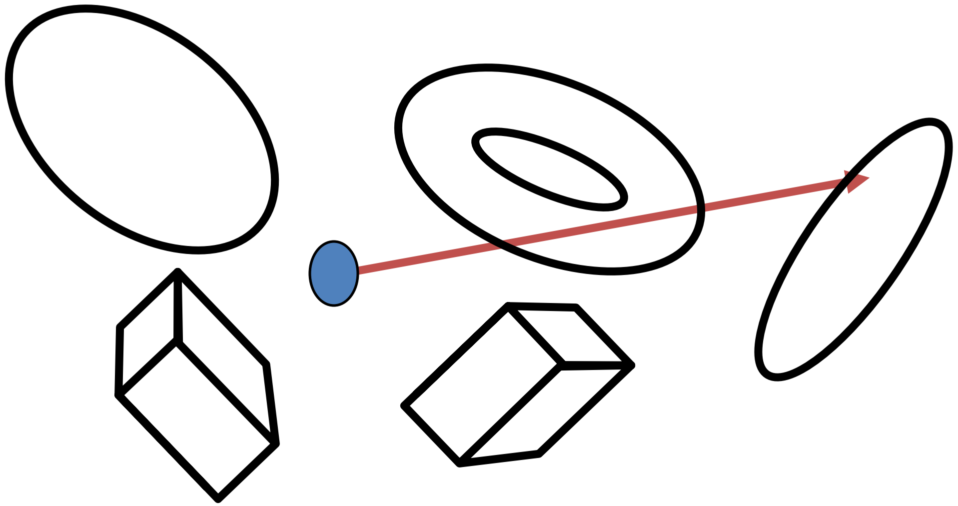
(d) HAIRBALL (2.9M triangles)

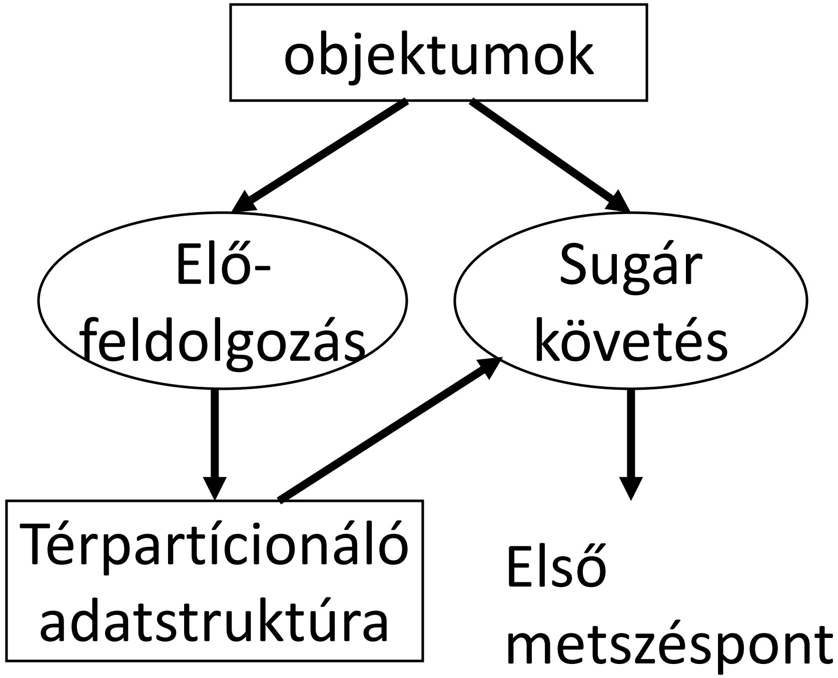


(e) POWER PLANT (12.7M triangles)

(f) SAN MIGUEL (10.5M triangles)

Térpartícionáló módszerek

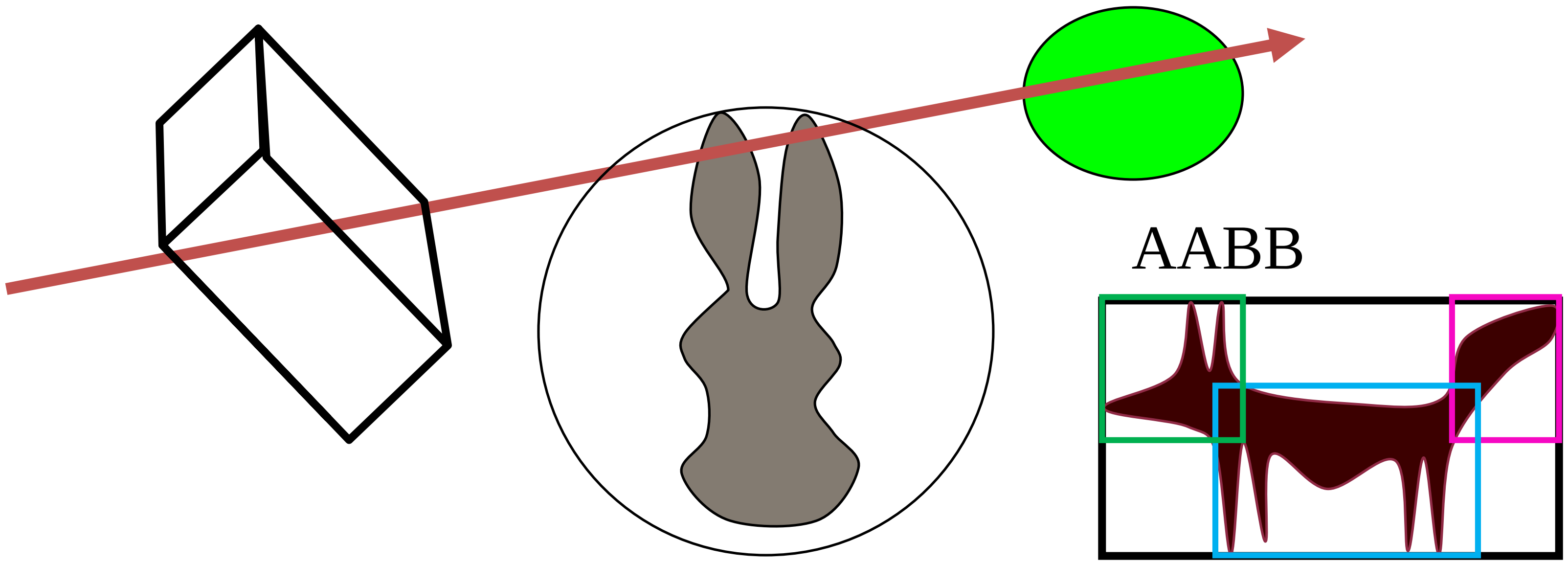




Adatstruktúra:

- Ha ismert a sugár, akkor a potenciális metszett objektumok számát csökkenti
- Ha a potenciálisak közül találunk egyet, akkor a többi nem lehet közelebb

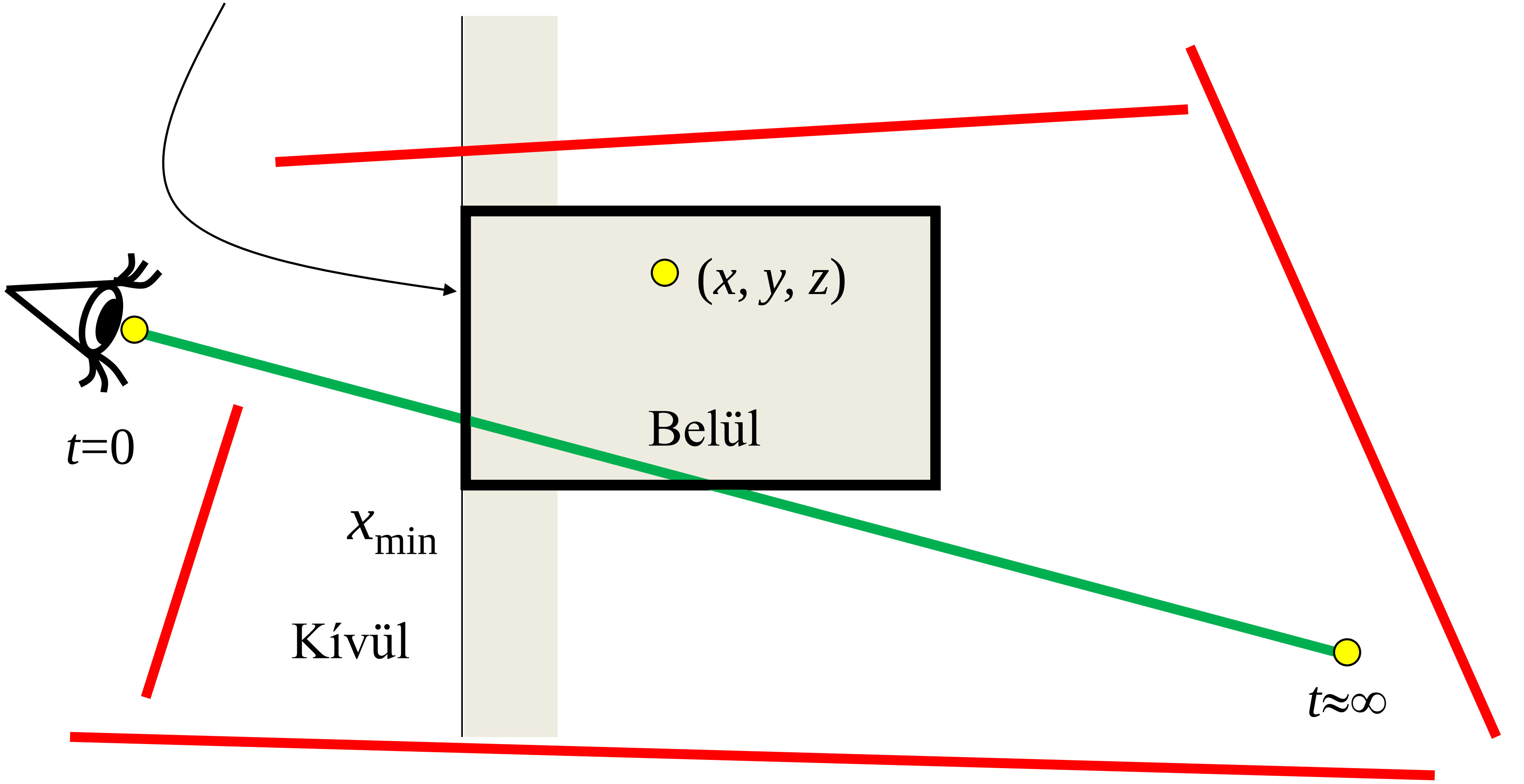
Befoglaló térfogat (Bounding Volume)



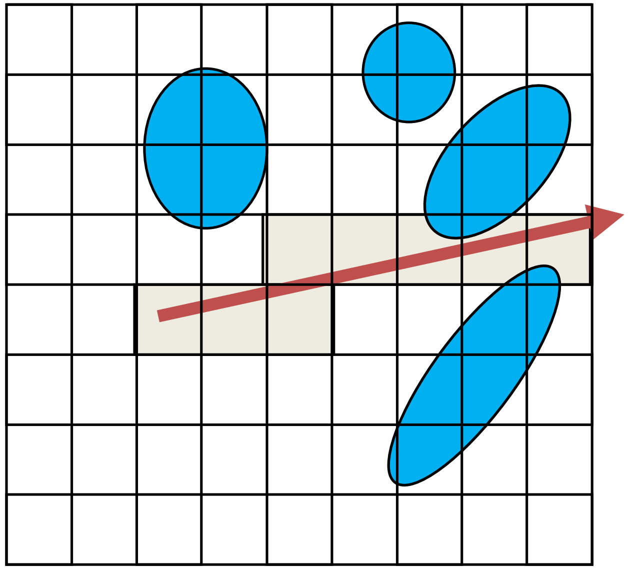

```
double IntersectBV( ray, object )          // < 0 ha nincs
    IF ( Intersect( ray, bounding volume of object) < 0) RETURN -1;
    RETURN Intersect( ray, object );
END
```

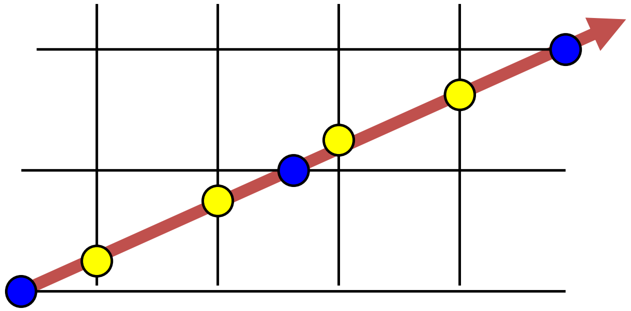
AABB metszés vágással

$x > x_{min}$ $x < x_{max}$ $y > y_{min}$ $y < y_{max}$ $z > z_{min}$ $z < z_{max}$



Reguláris térháló





Előfeldolgozás:

Minden cellára a metszett objektumok

komplexitás: $O(n \cdot c) = O(n^2)$

Sugárkövetés:

FOR each cell of the line // line drawing

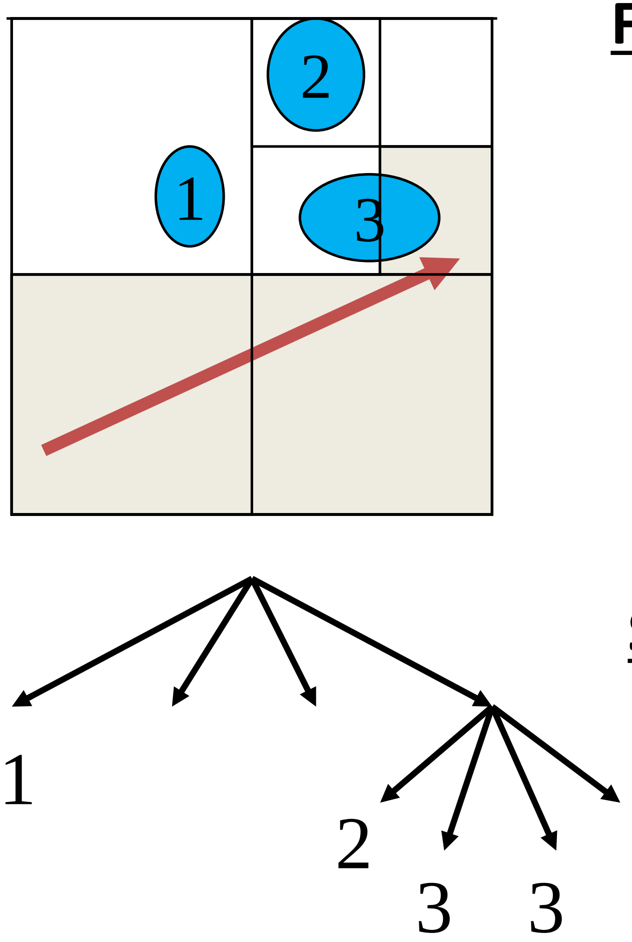
Metszés a cellában lévőkkel

IF van metszés RETURN

ENDFOR

átlagos eset komplexitás: $O(1)$

Nyolcas (oktális) fa



Octree

<u>Faépítés(cella):</u>

IF a cellában kevés objektum van

  cellában regisztráld az objektumokat

ELSE

  cellafelezés: c1, c2, …, c8

  Faépítés(c1); … Faépítés(c8);

ENDIF

<u>Sugárkövetés:</u>

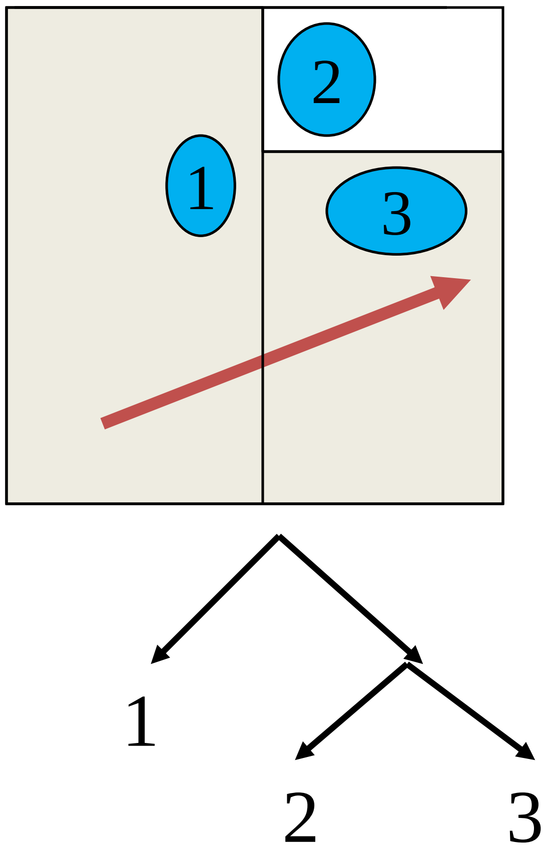
FOR összes sugár által metszett cellára

  Metszés a cellában lévőkkel

  IF van metszés RETURN

ENDFOR

Binary Space Partitioning fa (kd-fa)



kd-tree

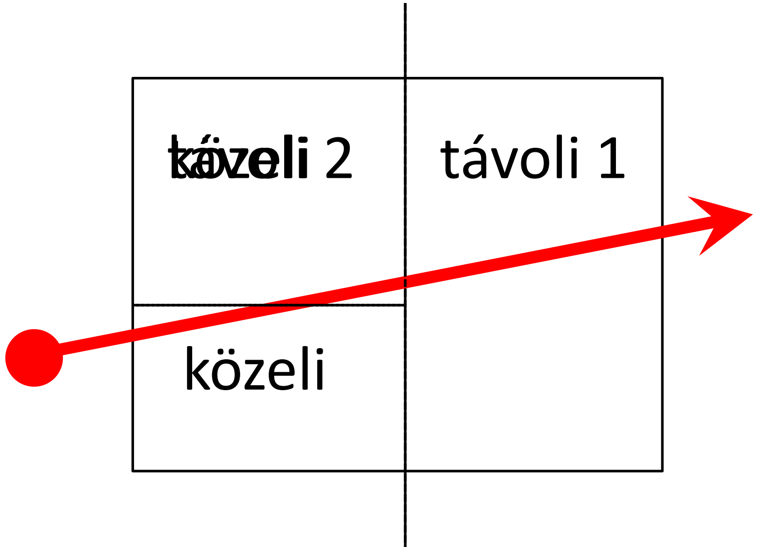
Faépítés(cella):

```
IF a cellában kevés objektum van
    cellában regisztráld az objektumokat
ELSE
    sík keresés
    cella felezés a síkkal: c1, c2
    Faépítés(c1); Faépítés(c2);
ENDIF
```

Sugárkövetés:

```
FOR each cell intersecting the line
    Metszés a cellában lévőkkel
    IF van metszés RETURN
ENDFOR
```

kd-fa bejárása



Befoglaló AABB
távoli 2
távoli 1

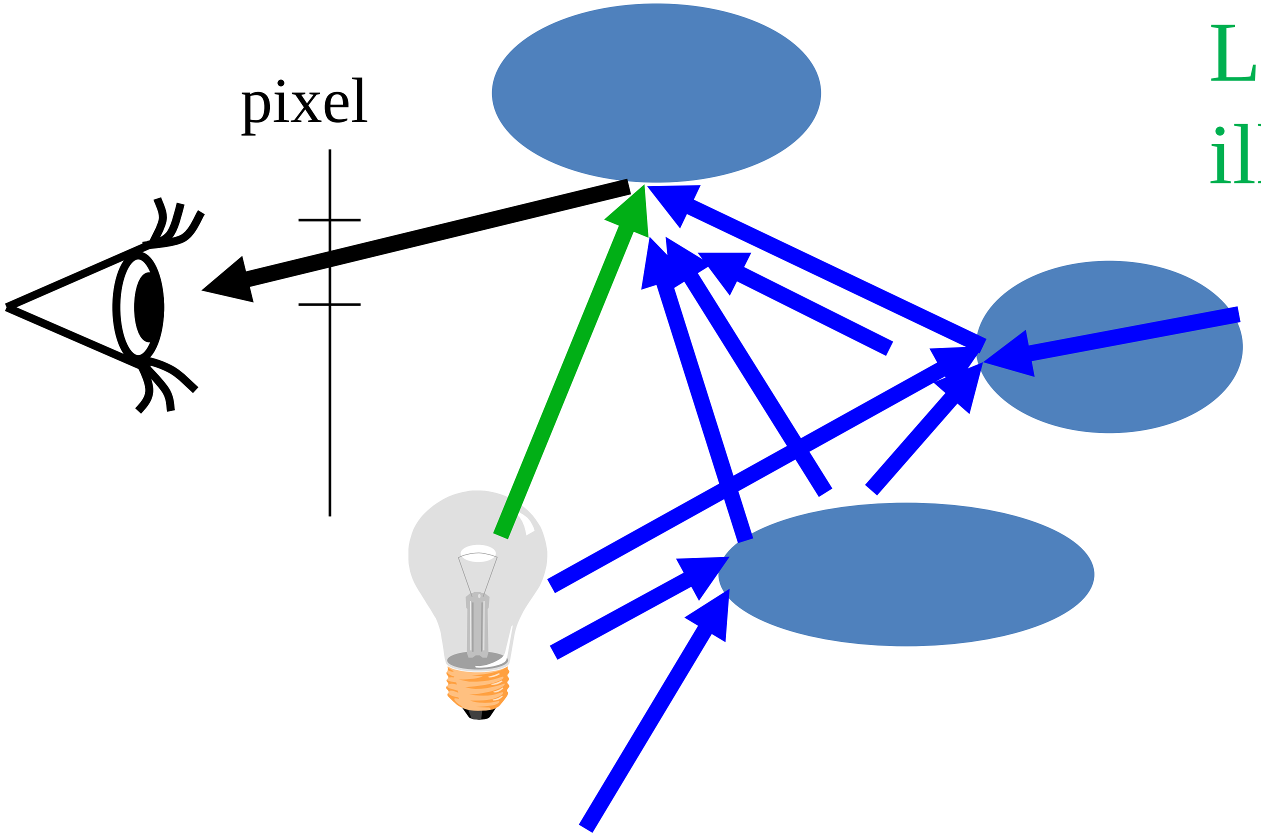
```
while (stack nem üres) {
    Pop(AABB)
    while (nem levél) {
        Ha van AABB-nek távoli
            Push(távoli)
        AABB = közeli
    }
    Regisztrált objektumok metszése
    Ha van metszéspont return
}
```

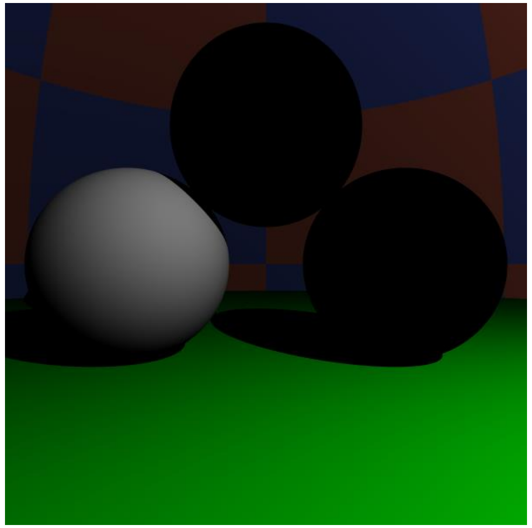
“*Úgy szeretnék integrálni.*”
Kockaéder

Globális illumináció: Path tracing

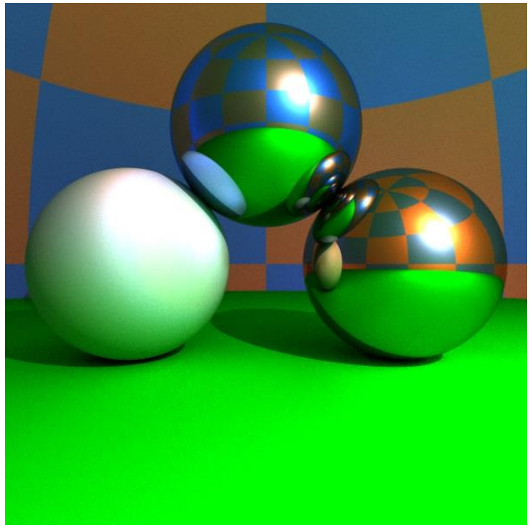
Szirmay-Kalos László

Képszintézis





Local illumination

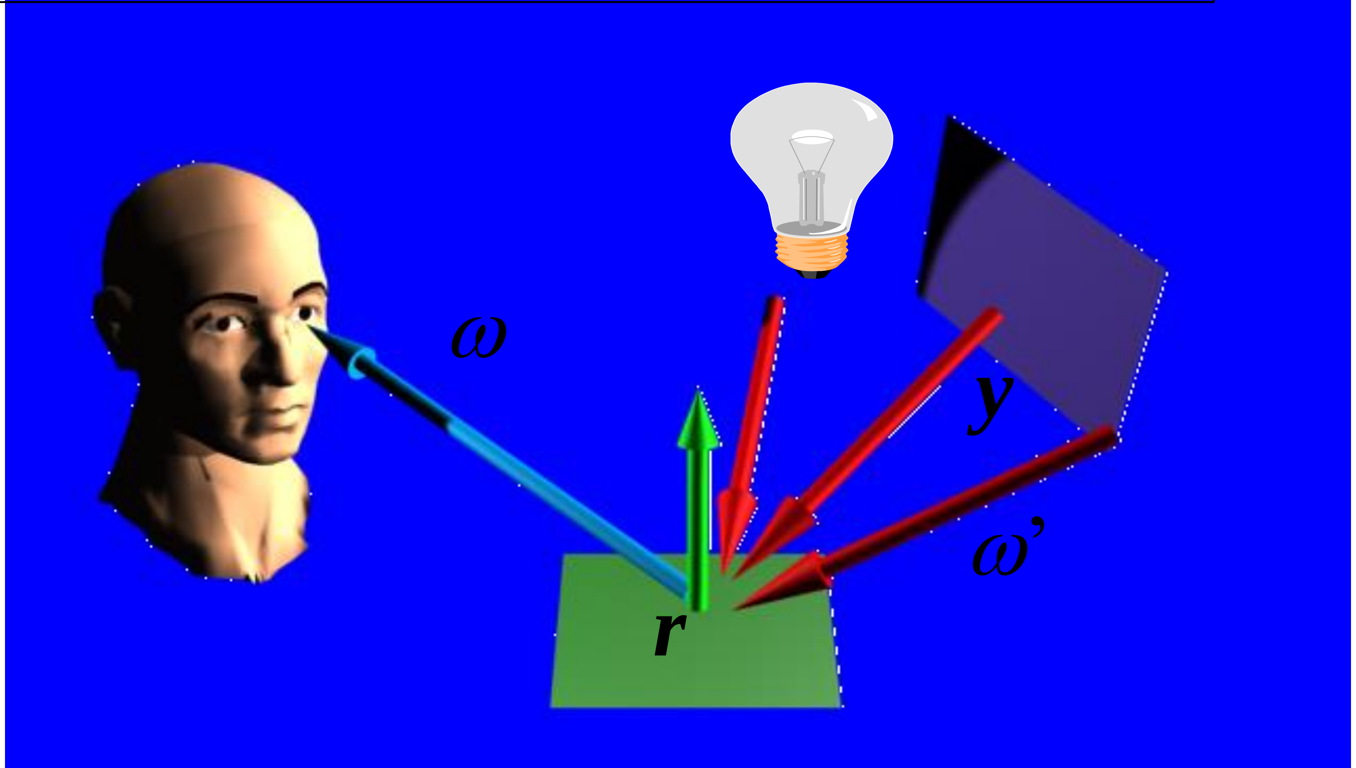


Global illumination

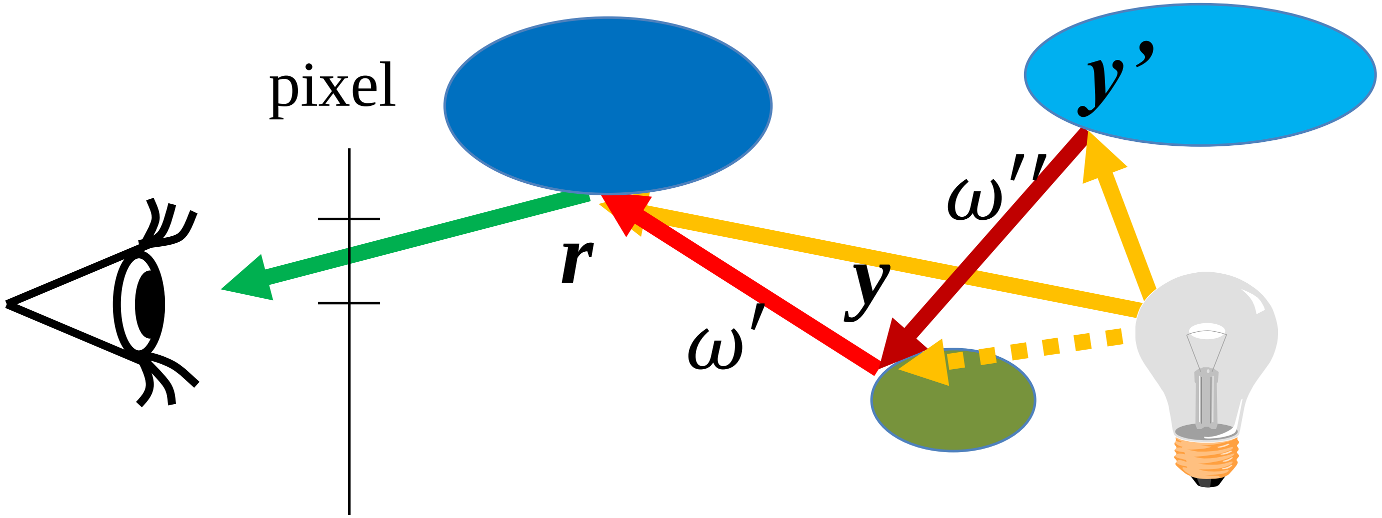
Rendering equation

OutRad = DirectLight + $\sum$ InRad * Reflection

$$L(\boldsymbol{r},\omega) = D(\boldsymbol{r},\omega) + \int_{\Omega} L(\boldsymbol{y},\omega')R(\omega,\omega')\,\mathrm{d}\omega'$$



A megoldás integrálok sorozata

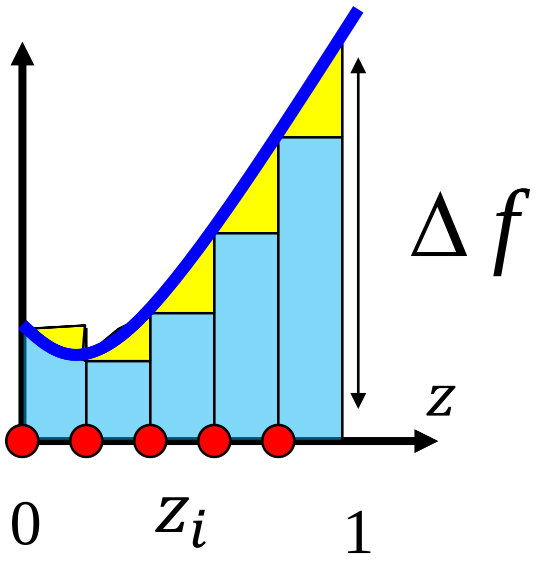


$$L(\boldsymbol{y}',\omega'') = D(\boldsymbol{y}',\omega'') + \int_{\Omega''} L(\boldsymbol{y}'',\omega''')R(\omega'',\omega''')\,\mathrm{d}\omega'''$$

$$L(\boldsymbol{y},\omega') = D(\boldsymbol{y},\omega') + \int_{\Omega'} L(\boldsymbol{y}',\omega'')R(\omega',\omega'')\,\mathrm{d}\omega''$$

$$\boxed{L(\boldsymbol{r},\omega) = D(\boldsymbol{r},\omega) + \int_{\Omega} L(\boldsymbol{y},\omega')R(\omega,\omega')\,\mathrm{d}\omega'}$$

Numerikus integrálás



$$\int_0^1 f(z)\mathrm{d}z \approx \frac{1}{M}\sum_{i=1}^{M} f(z_i)$$

M minta

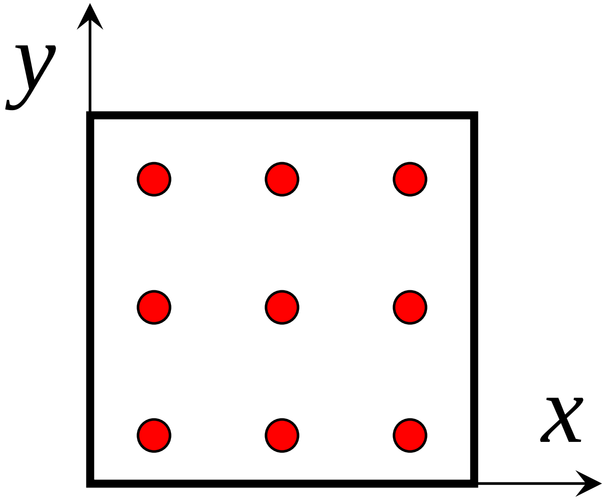
Átlagos magasság

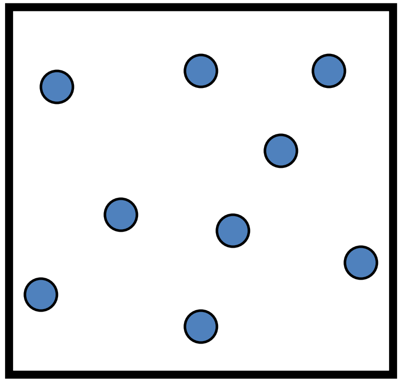
Alap

Háromszögek száma

$$\text{Error} = \frac{\Delta f}{2M} \cdot \frac{1}{M} \cdot M = \frac{\Delta f}{2M} = O(M^{-1})$$

Magasabb dimenziók: Megátkozva





Véletlen minták

$n = \sqrt{M}$ minta egy dimenzióban

Hiba 2-dim $= O(n^{-1}) = O(M^{-0.5})$

Hiba D-dim $= O(n^{-1}) = O(M^{-1/D})$

Monte Carlo integrálás:
Integrál = várható érték

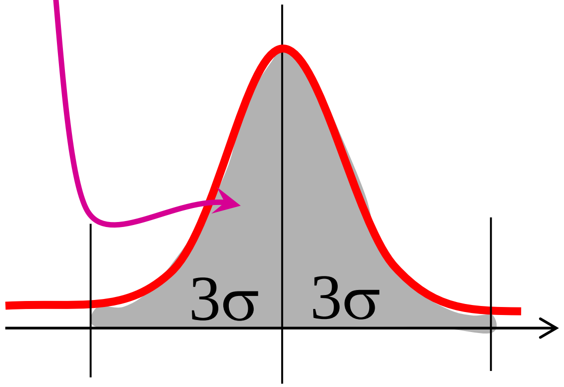
$$\int f(z)\mathrm{dz} = \int \frac{f(z)}{p(z)} p(z)\mathrm{dz} = \mathbf{E}\left[\frac{f(z)}{p(z)}\right] \approx \frac{1}{M}\sum_{i=1}^{M}\frac{f(z_i)}{p(z_i)}$$

A becslő egy valószínűségi változó:

$$\text{Variancia} = \sigma^2 = \mathbf{D}^2\left[\frac{f(z)}{p(z)}\right]\frac{1}{M}$$

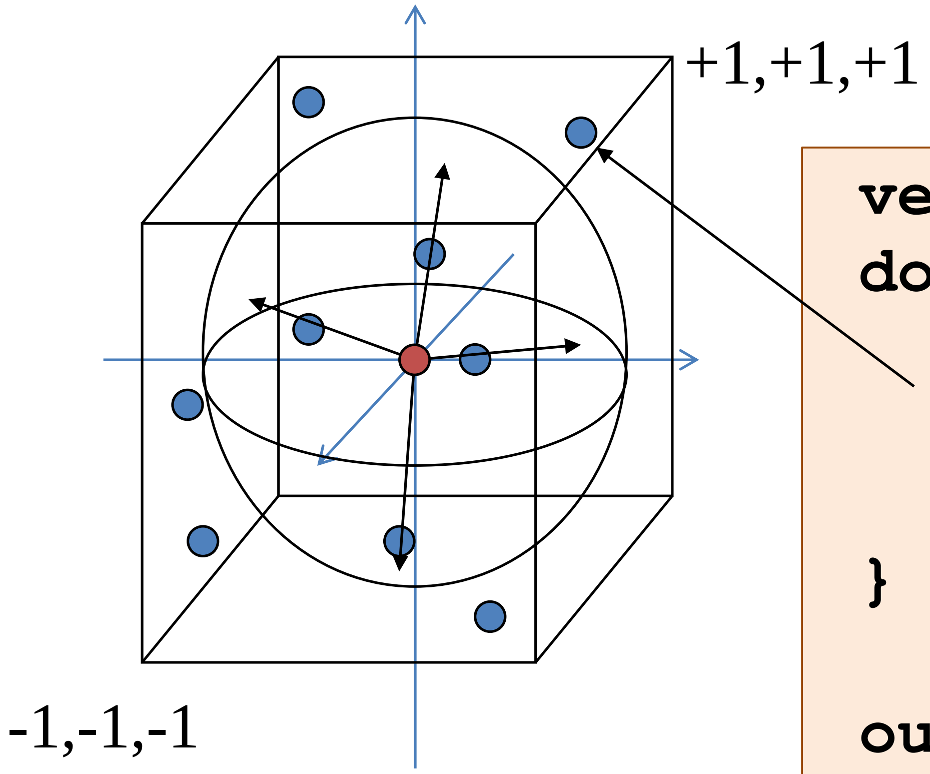
3 × szóráson belül
99.7% konfidenciával

$$\boxed{\text{Error} < 3\sigma = \frac{3}{\sqrt{M}}\mathbf{D}\left[\frac{f(z)}{p(z)}\right]}$$



Irányok generálása egyenletes valószínűséggel

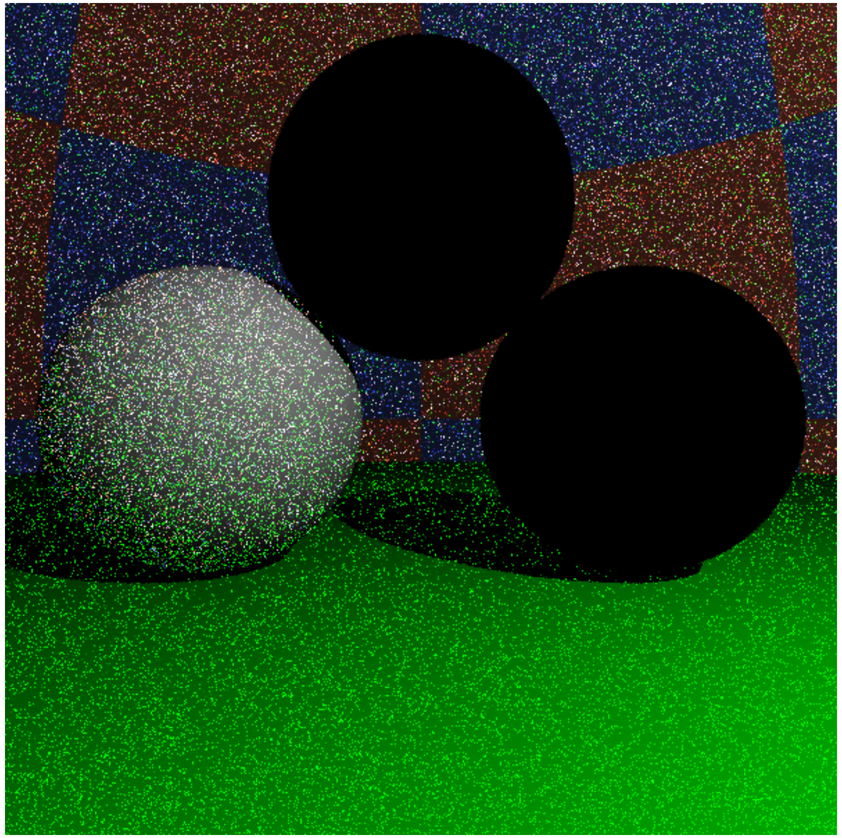
```
float random() { return (float)rand()/RAND_MAX; }
```



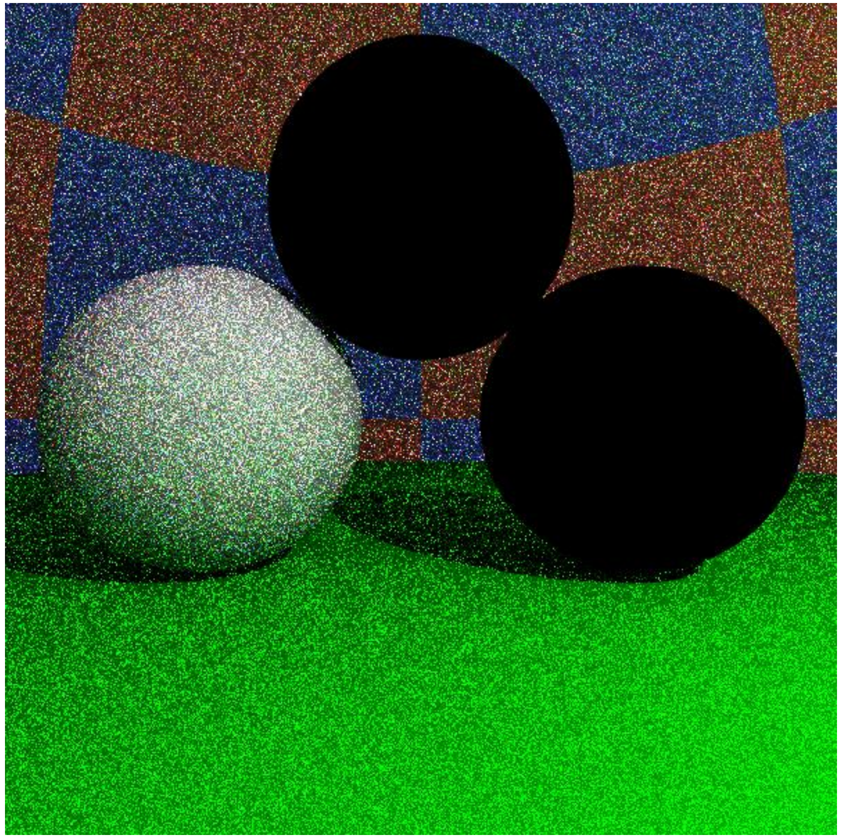

```
vec3 p;
do {
   p.x = 2*random()-1;
   p.y = 2*random()-1;
   p.z = 2*random()-1;
} while(dot(p, p) > 1);

outDir = normalize(p);
pdf = 1.0/4.0/M_PI;
```

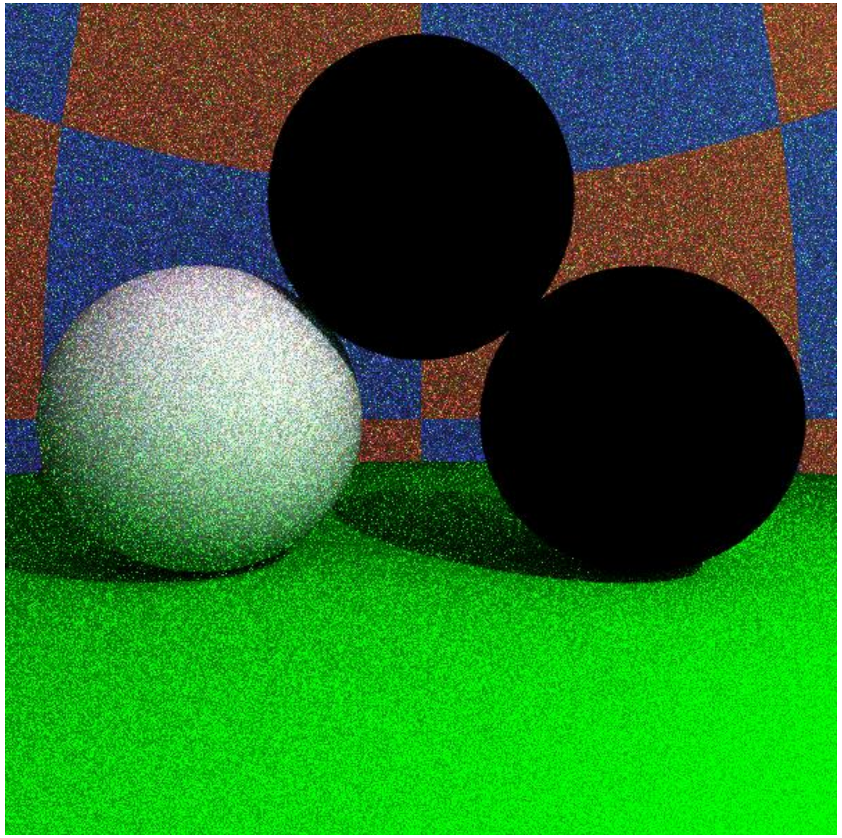
Uniform



1 sample/pixel

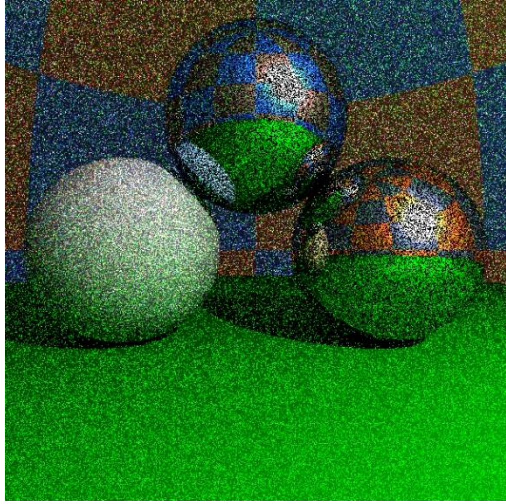


10 samples/pixel



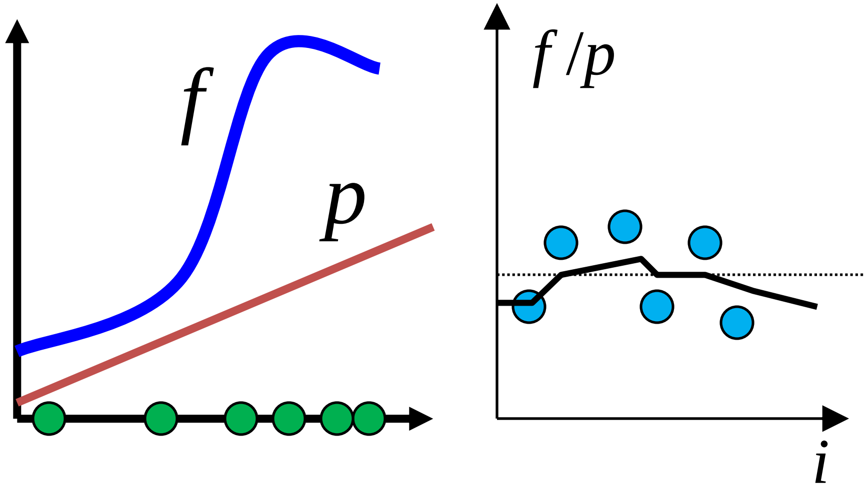
100 samples/pixel

Fontosság szerinti mintavétel



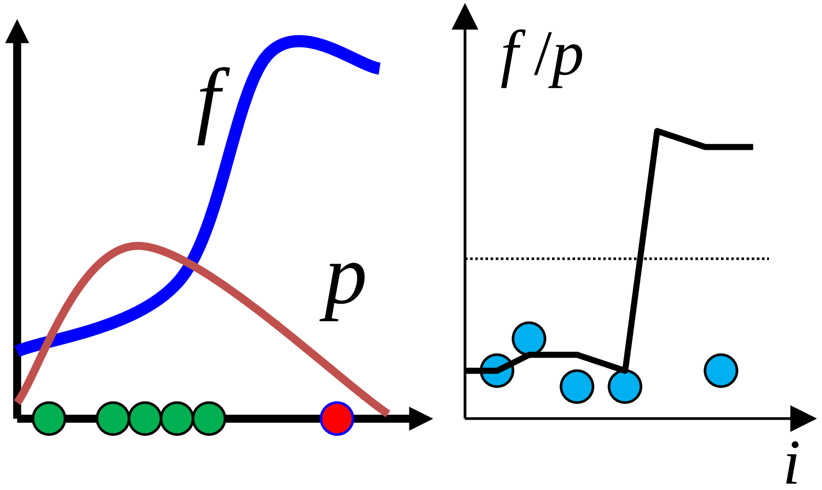
Becslő: $\frac{1}{M}\sum_{i=1}^{M}\frac{f(z_i)}{p(z_i)}$

$f(z)/p(z)$ legyen lapos. Ahol f nagy, p is legyen nagy



jó

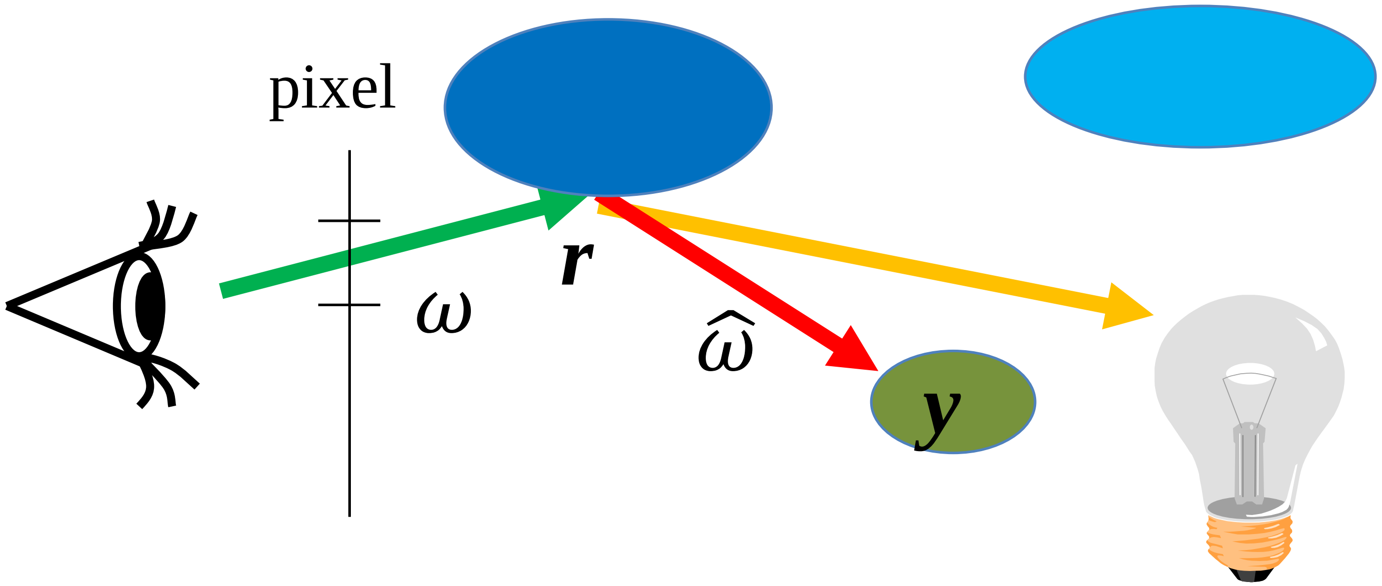
hasonló f/p hozzájárulások



rossz

Ritka, nagy f/p hozzájárulások

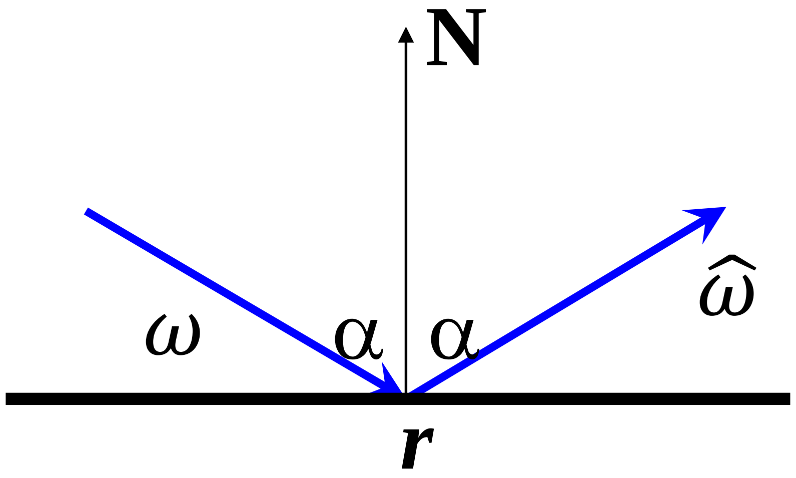
Véletlen bolyongás



$$\int_{\Omega} L(\boldsymbol{y}, \omega') R(\omega, \omega')\, \mathrm{d}\omega' \approx \frac{L(\boldsymbol{y}, \hat{\omega}) R(\omega, \hat{\omega})}{p(\hat{\omega})}$$

- $p(\hat{\omega})$ –nak az integrandusra kell hasonlítania (fontosság)
- A $L(\boldsymbol{y}, \omega')$ nem ismerjük, mert éppen számoljuk
- $p(\hat{\omega})$ legyen arányos $R(\omega, \hat{\omega})$

Tükör: Irány diszkrét eloszlású (Diract-delta)

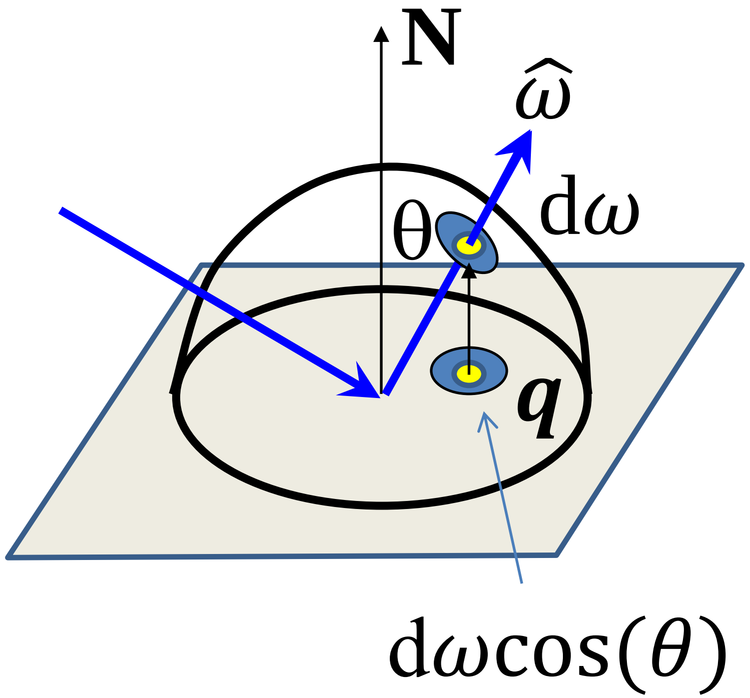


$$\cos\alpha = -(\omega\cdot\mathbf{N})$$

$$\boxed{\hat{\omega} = \omega + 2\mathbf{N}\cos\alpha}$$

```
float SampleMirror(vec3 N, vec3 inDir, vec3& outDir) {
   outDir = inDir - N * dot(N, inDir) * 2.0f;
   return 1; // pdf
}
```

Diffúz: Irány cos eloszlással



$$p(\hat{\omega})\, d\omega = \mathrm{Prob}\{\hat{\omega} \text{ in } d\omega\}$$

$$= \mathrm{Prob}\{\boldsymbol{q} \text{ in } d\omega\cos(\theta)\}$$

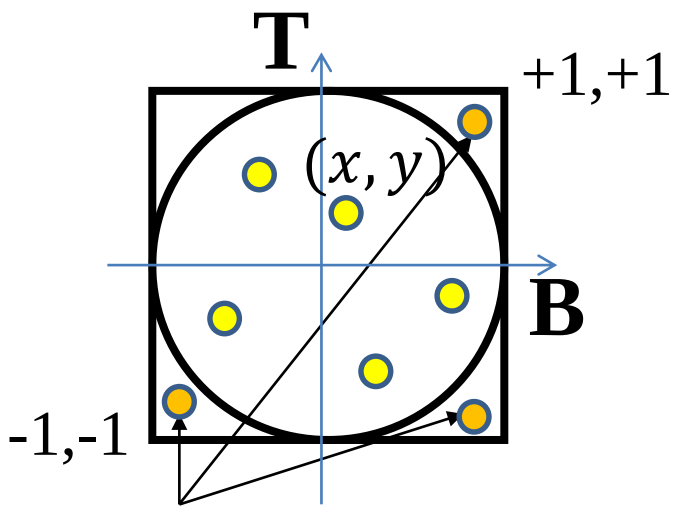
$$= \frac{d\omega\cos(\theta)}{\text{kör területe}}$$

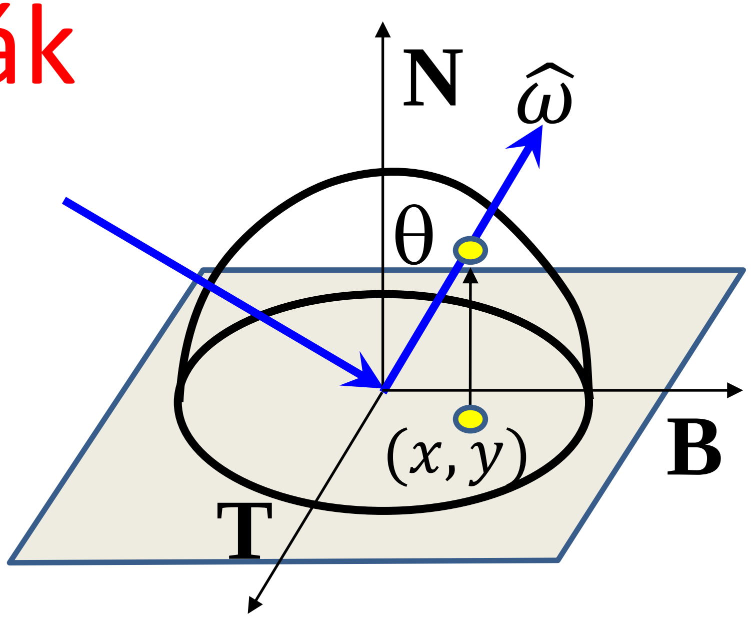
Ha $\boldsymbol{q}$ egyenletes eloszlású

$$= \frac{d\omega\cos(\theta)}{\pi}$$

$$\boxed{p(\hat{\omega}) = \frac{\cos(\theta)}{\pi}}$$

Cos eloszlású minták



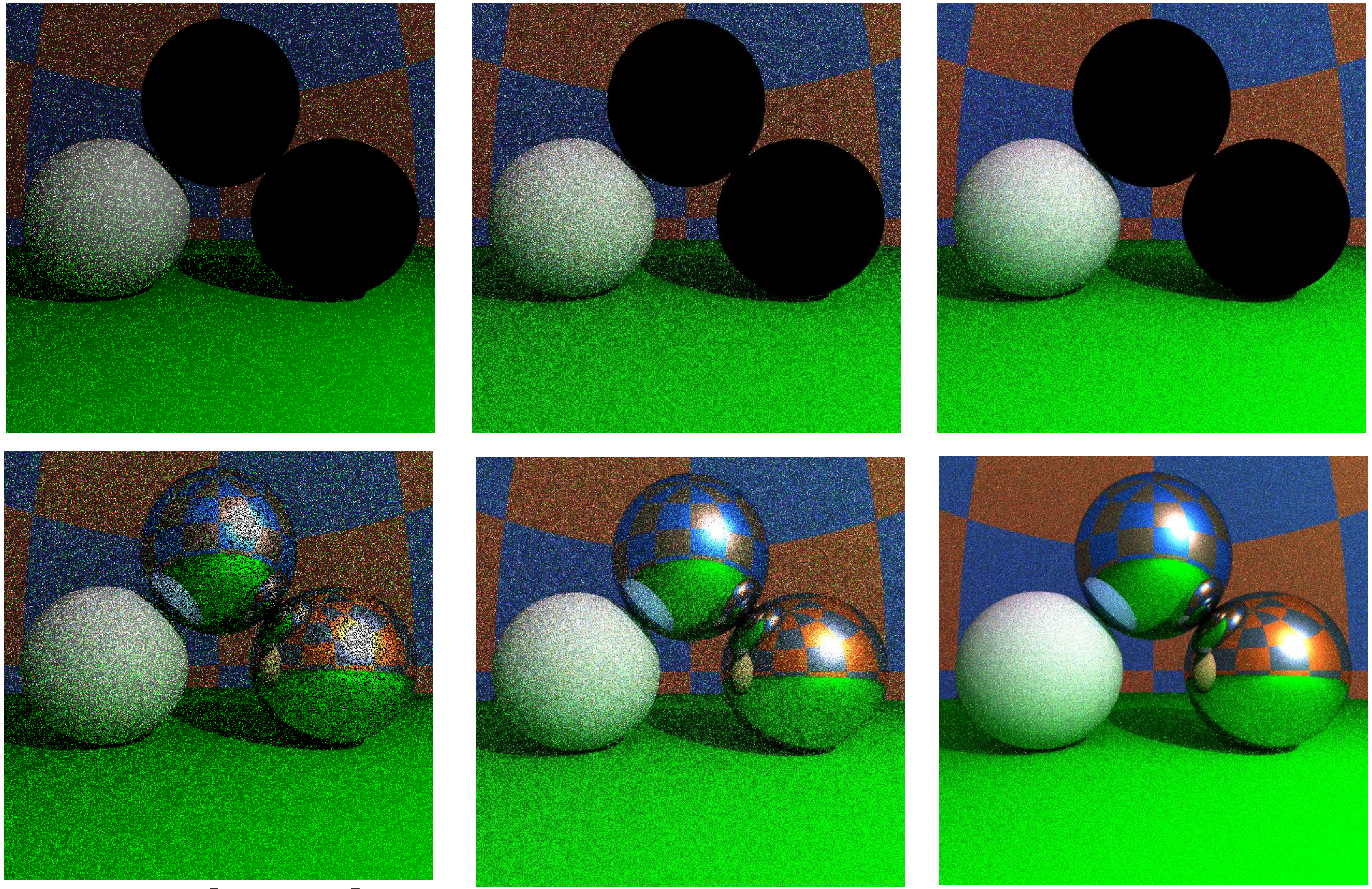



```
float SampleDiffuse(vec3 N, vec3 inDir, vec3& outDir) {
   vec3 T = normalize(cross(N, vec3(1,0,0)));
   vec3 B = cross(N, T);
   float x, y, z;
   do { x = 2*random()-1; y = 2*random()-1; } while(x*x+y*y>1);
   z = sqrtf(1 - x*x - y*y); // project to hemisphere
   outDir = T * x + B * y + N * z;
   return z/M_PI;  // pdf
}
```

Fontosság szerinti mintavétel

egyenletes

cos + diszkrét



1 sample/pixel | 10 samples/pixel | 100 samples/pixel

Megállás és visszaverődés típus: Orosz rulett

1. Monte Carlo integrál:

$$\int_\Omega L(\mathbf{y},\omega')R(\omega,\omega')\,\mathrm{d}\omega' = \mathbf{E}\left[\frac{L(\mathbf{y},\hat{\omega})k_d\cos^+(\theta)}{p_d(\hat{\omega})}\right] + \mathbf{E}\left[\frac{L(\mathbf{y},\hat{\omega})F\delta(\omega_m,\hat{\omega})}{p_m(\hat{\omega})}\right]$$

$R_d + R_m$ $\quad$ $\mathbf{E}[L_d]$ $\quad$ $\mathbf{E}[L_m]$

2. Számold $\mathbf{E}[L_d]$ -t s_d valószínűséggel, $\mathbf{E}[L_m]$ -t s_m valószínűséggel, egyébként 0
3. Kompenzálj s-sel osztással

Várható érték OK:

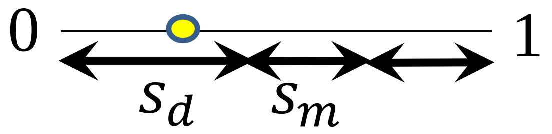
$$s_d\mathbf{E}[L_d/s_d] + s_m\mathbf{E}[L_m/s_m] + (1 - s_d - s_m)0 = \mathbf{E}[L_d] + \mathbf{E}[L_m]$$

A visszaverődés típusának kiválasztása

$$\frac{L_d}{s_d} = \frac{L(\boldsymbol{y},\hat{\omega})k_d \cos^+(\theta)}{p_d(\hat{\omega})s_d} = L(\boldsymbol{y},\hat{\omega})\frac{k_d \cos^+(\theta)}{\frac{\cos(\theta)}{\pi}s_d} = L(\boldsymbol{y},\hat{\omega})\frac{k_d\pi}{s_d}$$

$$\frac{L_m}{s_m} = \frac{L(\boldsymbol{y},\hat{\omega})F\delta(\omega_m,\hat{\omega})}{p_m(\hat{\omega})s_m} = L(\boldsymbol{y},\hat{\omega})\frac{F}{s_m}$$

- Tükörirány valószínűsége: $s_m = F$ luminance
- Diffúz irány valószínűsége: $s_d = k_d\pi$ luminance
- Megállás valószínűsége: $1 - s_m - s_d$



Path Tracer

```
vec3 trace(Ray ray, int depth = 0) {
   Hit hit = firstIntersect(ray);
   if (hit.t < 0 || depth >= maxdepth) return vec3(0,0,0);
   [r, N, k_d, n, κ] ← hit;
   vec3 outRad = DirectLight(hit);
   rnd = random();        // Russian roulette
   s_d = Luminance(k_d π); s_m = Luminance(Fresnel(ray.dir,N));
   if (rnd < s_d) { // diffuse
      pdf = SampleDiffuse(N, ray.dir, outDir);
      inRad = trace(Ray(r + Nε, outDir), depth+1);
      outRad += inRad * k_d * dot(N, outDir) / pdf / s_d;
   } else if (rnd < s_d + s_m) { // mirror
      pdf = SampleMirror(N, ray.dir, outDir);
      inRad = trace(Ray(r + Nε, outDir), depth+1);
      outRad += inRad * Fresnel(ray.dir,N) / pdf / s_m;
   }
   return outRad;
}
```