

*"In theory, there is no difference
between theory and practice.
In practice, there is."*

Benjamin Brewster

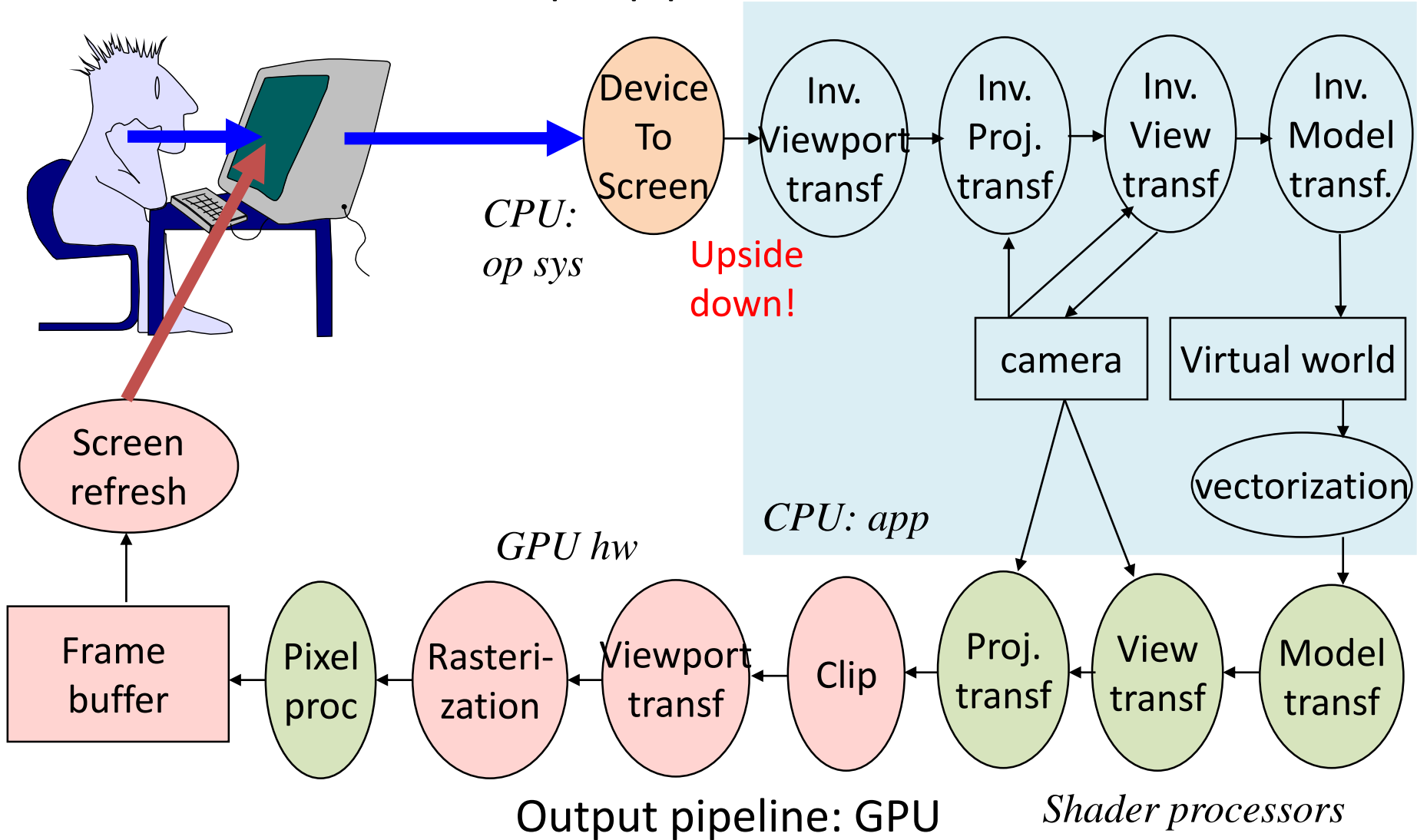
Graphics hardware and software

Szirmay-Kalos László

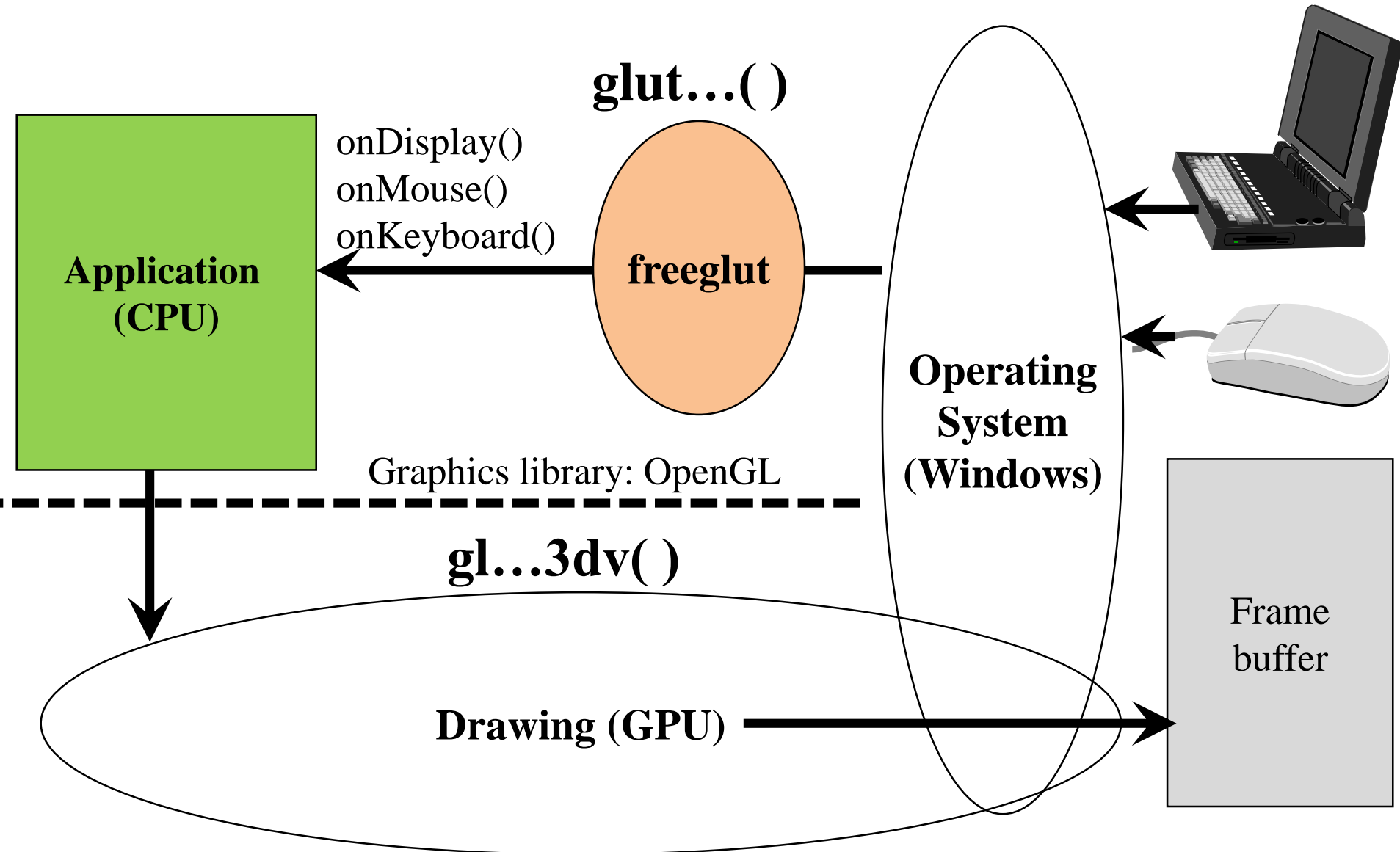


Interactive graphics systems

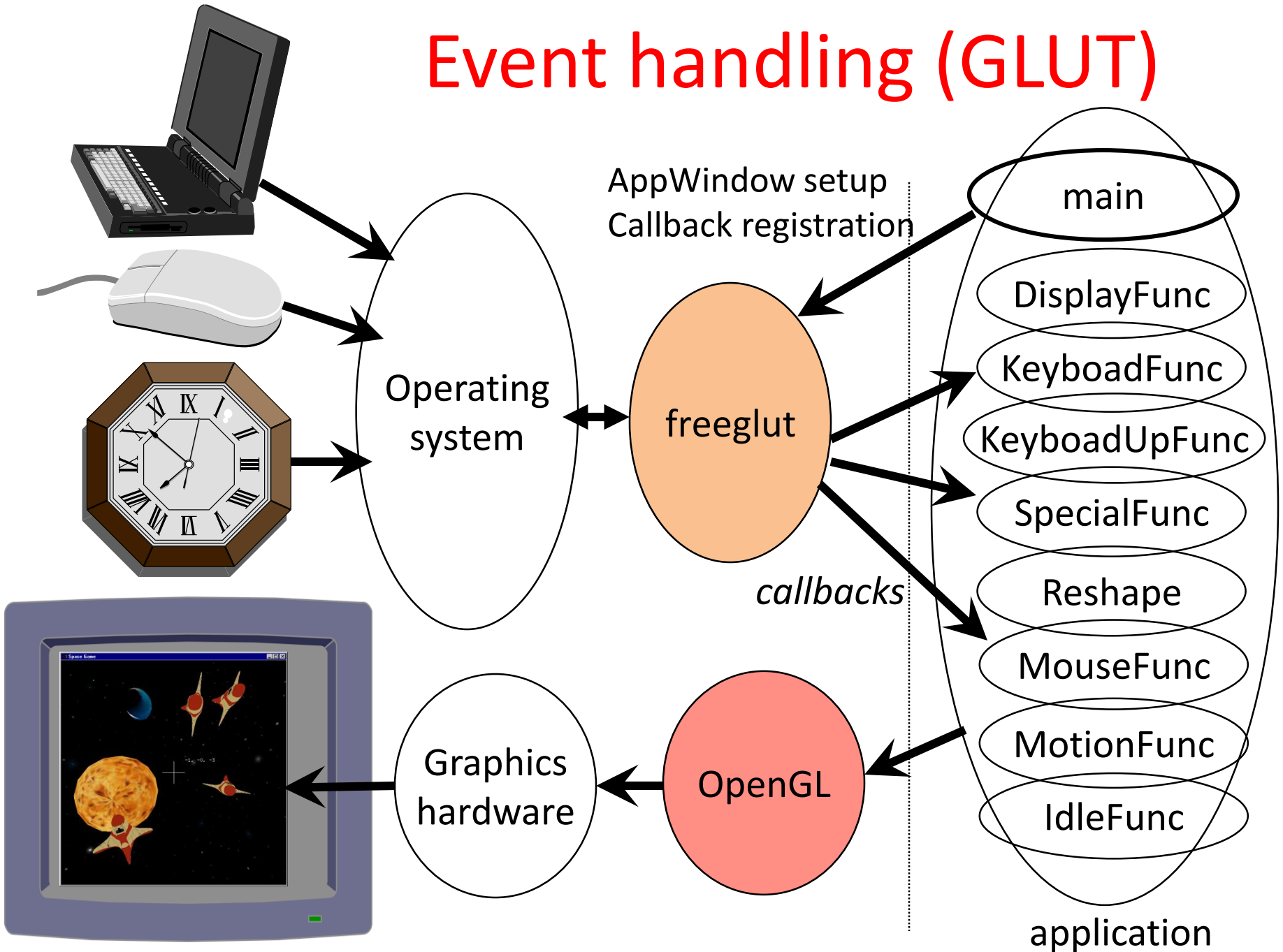
Input pipeline



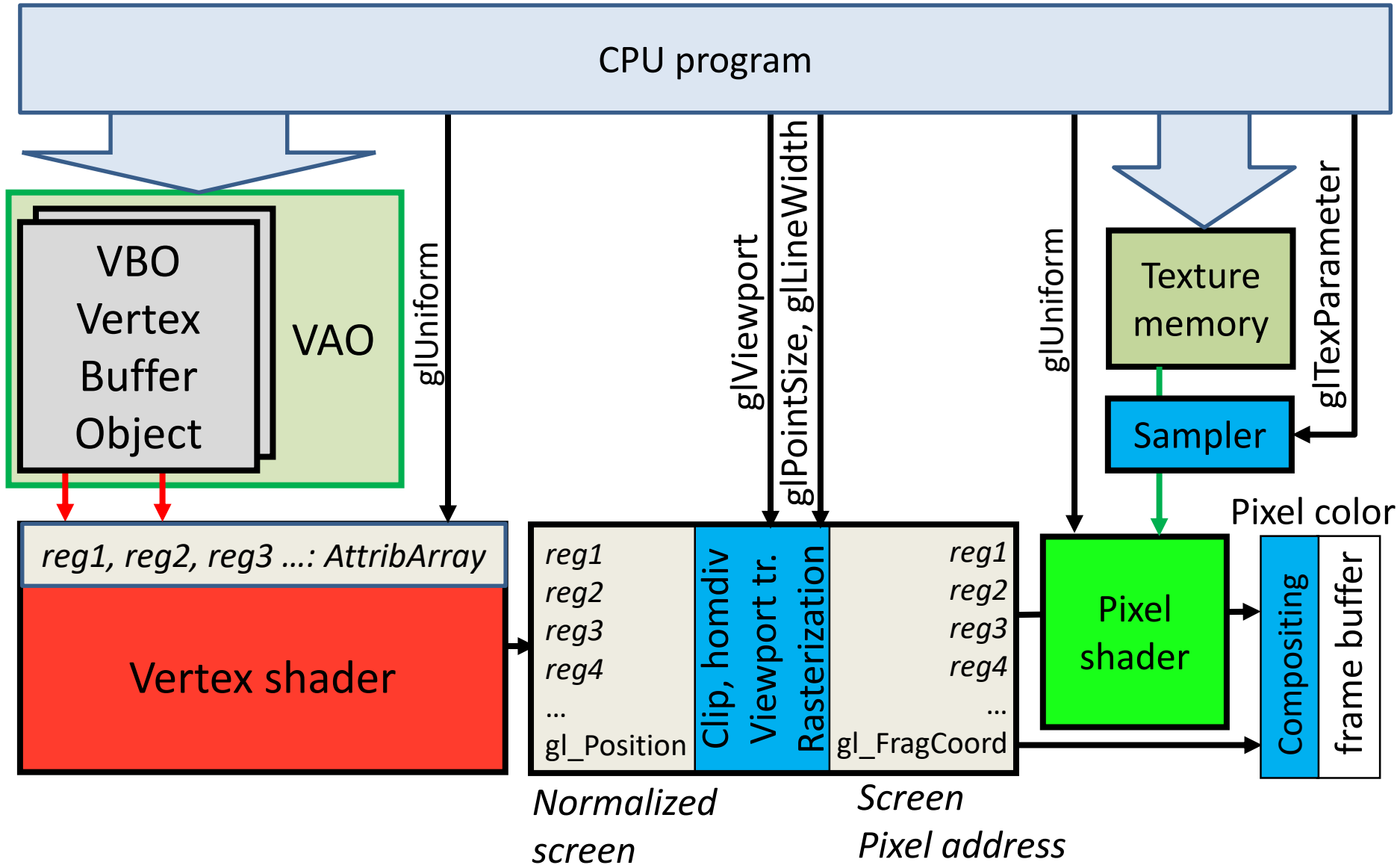
Software architecture



Event handling (GLUT)

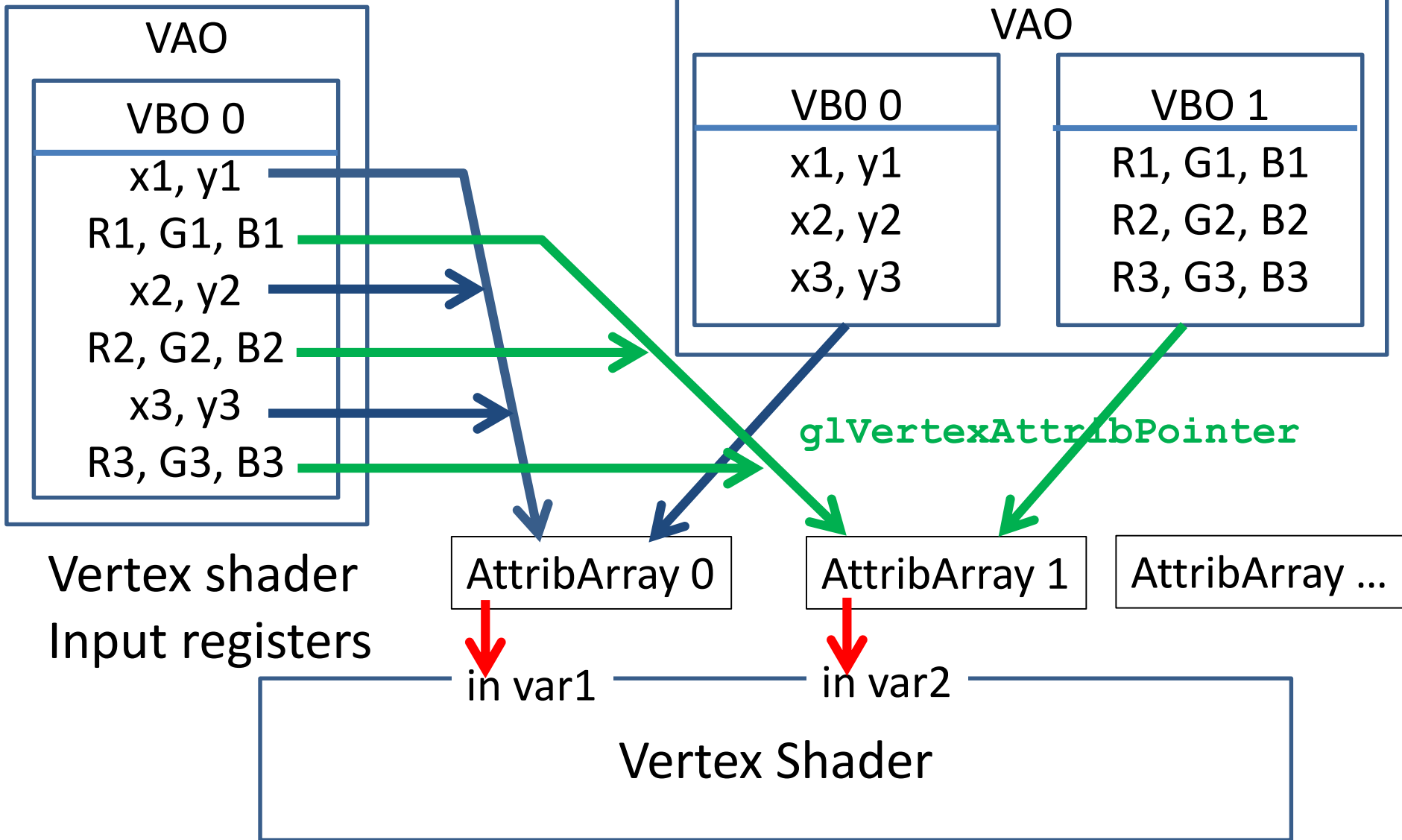


OpenGL 3.3 ... 4.6



Vertex data streaming

interleaved



My first OpenGL program

```
#include <windows.h>    // Only in MsWin
#include <GL/glew.h>    // download
#include <GL/freeglut.h> // download

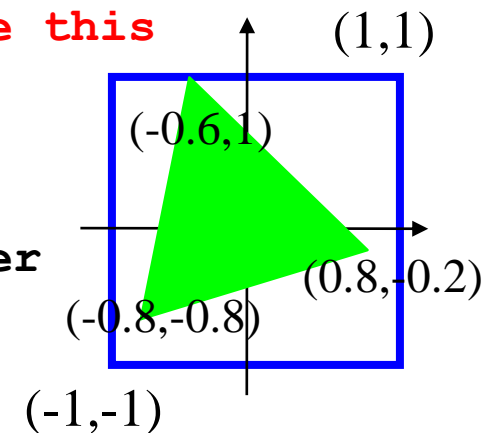
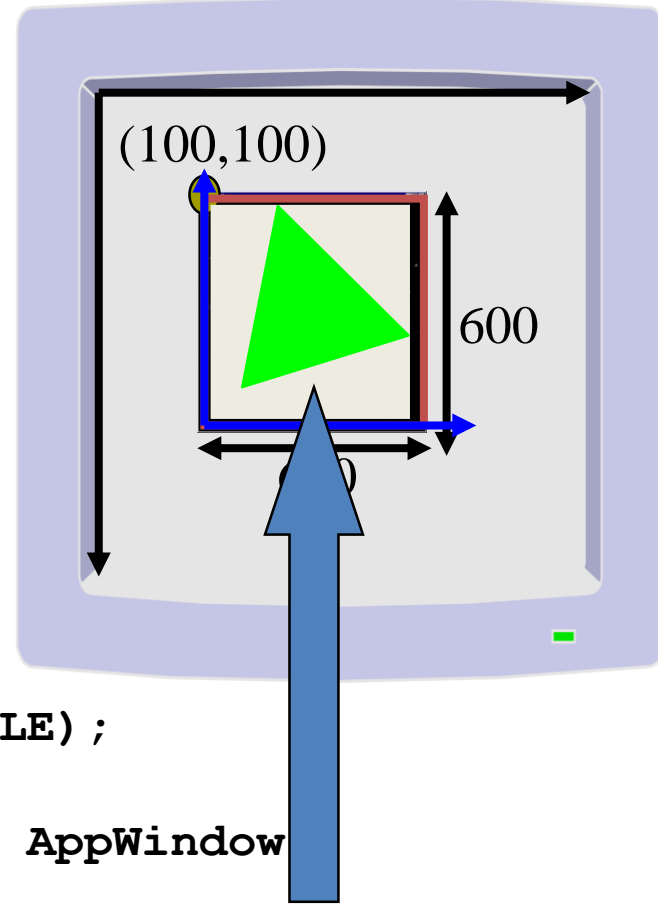
int main(int argc, char * argv[]) {
    glutInit(&argc, argv); // init glut
    glutInitContextVersion(3, 3);
    glutInitWindowSize(600, 600);
    glutInitWindowPosition(100, 100);
    glutInitDisplayMode(GLUT_RGBA|GLUT_DOUBLE);

    glutCreateWindow("Hi Graphics"); //show AppWindow
    glewExperimental = true; // magic
    glewInit(); // init glew, no opengl before this

    glViewport(0, 0, 600, 600); //photo
    onInitialization(); // next slide

    glutDisplayFunc(onDisplay); //event handler

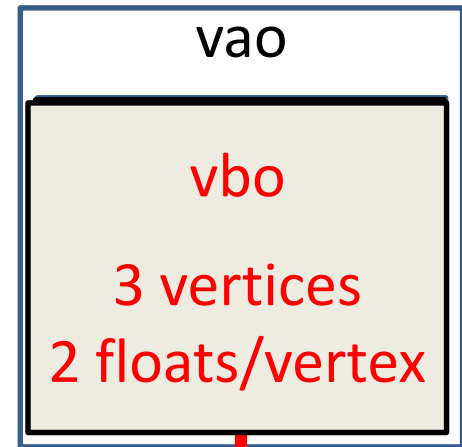
    glutMainLoop();
    return 1;
}
```



onInitialization()

```
unsigned int shaderProgram;  
unsigned int vao; // virtual world on the GPU
```

```
void onInitialization() {  
    glGenVertexArrays(1, &vao);  
    glBindVertexArray(vao); // make it active  
  
    unsigned int vbo; // vertex buffer object  
    glGenBuffers(1, &vbo); // Generate 1 buffer  
    glBindBuffer(GL_ARRAY_BUFFER, vbo);  
    // Geometry with 24 bytes (6 floats or 3 x 2 coordinates)  
    float vertices[] = {-0.8, -0.8, -0.6, 1.0, 0.8, -0.2};  
    glBufferData(GL_ARRAY_BUFFER, // Copy to GPU target  
                sizeof(vertices), // # bytes  
                vertices, // address  
                GL_STATIC_DRAW); // we do not change later  
    glEnableVertexAttribArray(0); // AttribArray 0  
    glVertexAttribPointer(0, // vbo -> AttribArray 0  
                          2, GL_FLOAT, GL_FALSE, // two floats/attrib, not fixed-point  
                          0, NULL); // stride, offset: tightly packed  
}
```



AttribArray 0


```
#version 330
precision highp float;
uniform mat4 MVP;
layout(location = 0) in vec2 vp; //AttribArray 0

void main() {
    gl_Position = vec4(vp.x, vp.y, 0, 1) * MVP;
}
```

```
#version 330
precision highp float;
uniform vec3 color;
out vec4 outColor;

void main() {
    outColor = vec4(color, 1);
}
```

```
static const char * vertexSource = R"( ... )
static const char * fragmentSource = R"( ... )" ;
unsigned int vertexShader = glCreateShader(GL_VERTEX_SHADER);
glShaderSource(vertexShader, 1, &vertexSource, NULL);
glCompileShader(vertexShader);

unsigned int fragmentShader=glCreateShader(GL_FRAGMENT_SHADER);
glShaderSource(fragmentShader, 1, &fragmentSource, NULL);
glCompileShader(fragmentShader);

shaderProgram = glCreateProgram();
glAttachShader(shaderProgram, vertexShader);
glAttachShader(shaderProgram, fragmentShader);

glBindFragDataLocation(shaderProgram, 0, "outColor");

glLinkProgram(shaderProgram);
glUseProgram(shaderProgram);
}
```

```
uniform mat4 MVP;
layout(location = 0) in vec2 vp;

void main() {
    gl_Position = vec4(vp.x, vp.y, 0, 1) * MVP;
}
```

```
uniform vec3 color;
out vec4 outColor;

void main() {
    outColor = vec4(color, 1);
}
```

```
void onDisplay( ) {
    glClearColor(0, 0, 0, 0); // background color
    glClear(GL_COLOR_BUFFER_BIT); // clear frame buffer

    // Set vertexColor to (0, 1, 0) = green
    int location = glGetUniformLocation(shaderProgram, "color");
    glUniform3f(location, 0.0f, 1.0f, 0.0f); // 3 floats

    float MVPtransf[4][4] = { 1, 0, 0, 0, // MVP matrix,
                              0, 1, 0, 0, // row-major!
                              0, 0, 1, 0,
                              0, 0, 0, 1 };

    location = glGetUniformLocation(shaderProgram, "MVP");
    glUniformMatrix4fv(location, 1, GL_TRUE, &MVPtransf[0][0]);

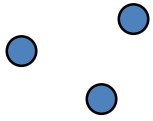
    glBindVertexArray(vao); // Draw call
    glDrawArrays(GL_TRIANGLES, 0 /*startIdx*/, 3 /*# Elements*/);

    glutSwapBuffers( ); // exchange buffers for double buffering
}
```

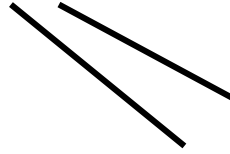
row-major

OpenGL primitives

```
glDrawArrays( primitiveType,  
              startIdx,  
              numElements);
```

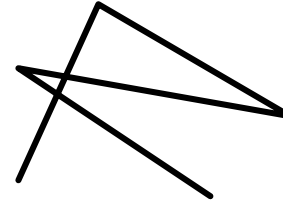


GL_POINTS

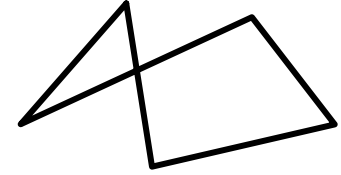


GL_LINES

Vectorized parametric curve

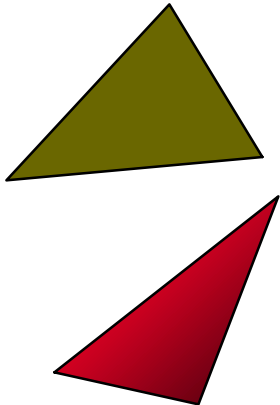


GL_LINE_STRIP



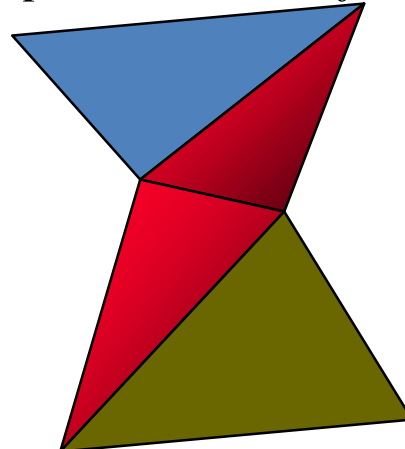
GL_LINE_LOOP

Output of Ear clipping



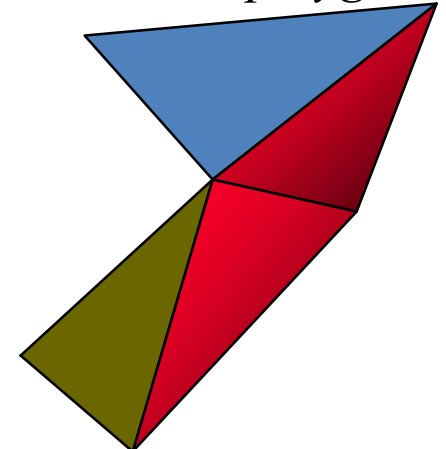
GL_TRIANGLES

*Tessellated
parametric surface*



GL_TRIANGLE_STRIP

„Convex” polygon



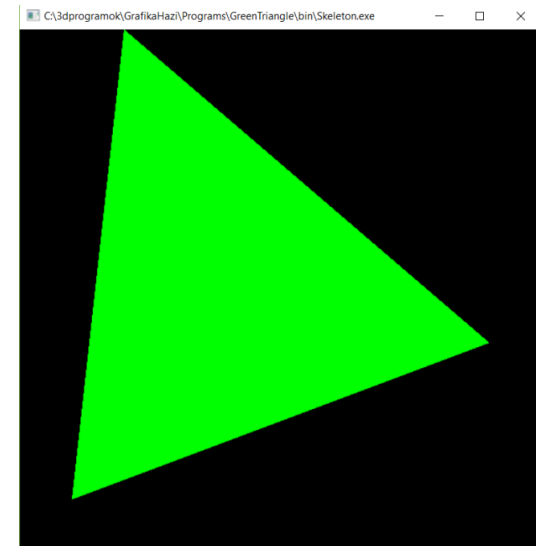
GL_TRIANGLE_FAN



Graphics hardware and software

Program: Framework and green triangle

Szirmay-Kalos László



OpenGL starters' kit: Shader programs

- `glCreate[Shader|Program]()` creation
- `glShaderSource()` source upload
- `glCompileShader()` compilation
- `glAttachShader()` add shader to program
- `glBindFragDataLocation()` what goes to frame buffer?
- `glLinkProgram()` linking
- `glUseProgram()` select for execution
- `glGetUniformLocation()` uniform variable address
- `glUniform*()` uniform variable set value

```
class GPUProgram {
    bool create(char * vertShader, char * fragShader,
               char * OutputName, char * geomShader = 0);
    void Use();
    void setUniform(...);
};
```

OpenGL starters' kit

Resource generation, binding, uploading

- `glGen[VertexArrays|Buffers|Textures](1, &id);`
- `glBind[VertexArray|Buffer|Texture](id);`
- `glBufferData(GL_ARRAY_BUFFER, ...);`
- `glTexImage2D(GL_TEXTURE_2D, ...);`

Connect VBO to input registers

- `glEnableVertexAttribArray(reg)` enable register
- `glVertexAttribPointer(reg, ...)` buffer to register

Drawing and pipeline management

- `glDrawArrays(type, start, num)` draw call
- `glClearColor(r, g, b, a)` background clear color
- `glClear(buffer)` clear background
- `glViewport(vx, vy, vw, vh)` viewport
- `glPointSize(s)` point size
- `glLineWidth(w)` line width

framework.h

```
include: <stdio.h>, <stdlib.h>, <math.h>, <vector>, <string>
        if windows <windows.h>
            <GL/glew.h>, <GL/freeglut.h> // must be downloaded

const unsigned int windowWidth = 600, windowHeight = 600;

struct vec2;
struct vec3;
struct vec4;
struct mat4;

struct Texture {
    unsigned int textureId;
    void create(...);
};

class GPUProgram {
    bool create(char * vertShader,
               char * fragShader, char * OutputName,
               char * geomShader = nullptr);

    void Use();
    void setUniform(...);
};
```

framework.cpp

```
#include "framework.h"

void onInitialization(); // Init
void onDisplay(); // Redraw
void onKeyboard(unsigned char key, int pX, int pY); // Key pressed
void onKeyboardUp(unsigned char key, int pX, int pY); // Key released
void onMouseMotion(int pX, int pY); // Move mouse with key pressed
void onMouse(int button, int state, int pX, int pY); // Mouse click
void onIdle(); // Time elapsed

int main(int argc, char * argv[]) {
    glutInit(&argc, argv); glutInitContextVersion(3, 3);
    glutInitWindowSize(windowWidth, windowHeight);
    glutInitWindowPosition(100, 100);
    glutInitDisplayMode(GLUT_RGBA | GLUT_DOUBLE | GLUT_DEPTH);
    glutCreateWindow(argv[0]);

    glewExperimental = true; glewInit(); // no opengl calls before this

    onInitialization();
    glutDisplayFunc(onDisplay); // Register event handlers
    glutMouseFunc(onMouse);
    glutIdleFunc(onIdle);
    glutKeyboardFunc(onKeyboard);
    glutKeyboardUpFunc(onKeyboardUp);
    glutMotionFunc(onMouseMotion);
    glutMainLoop(); return 1;
}
```


Skeleton.cpp

```
#include "framework.h"

const char * const vertexSource;
const char * const fragmentSource;

GPUProgram gpuProgram; // vertex and fragment shaders

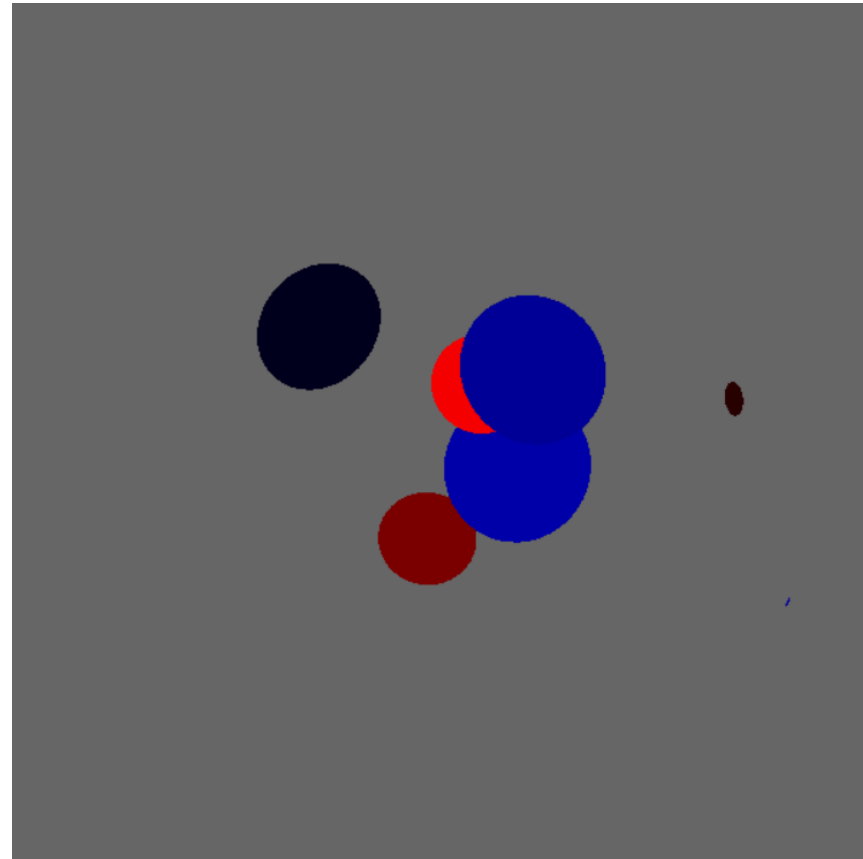
void onInitialization() {
    ...
    gpuProgram.create(vertexSource, fragmentSource, "outColor");
}

void onDisplay() {
    glClearColor(0, 0, 0, 0); // background color
    glClear(GL_COLOR_BUFFER_BIT); // clear frame buffer
    ...
    glutSwapBuffers(); // exchange buffers for double buffering
}

void onKeyboard(unsigned char key, int pX, int pY) { ... }
void onKeyboardUp(unsigned char key, int pX, int pY) { ... }
void onMouseMotion(int pX, int pY) { ... }
void onMouse(int button, int state, int pX, int pY) { ... }
void onIdle() { ... }
```

Charges in Soup

Homework 1



Specification

Particles of random charge and mass swim in a fluid of no charge in 2D Euclidean space. Particles are circles in this space, where the radius is proportional to the mass, the intensity is proportional to the absolute value of the charge, the hue is red for positive and green or blue for negative charges. Particles are moving according to the 2D Coulomb force and a soup friction that is proportional to the velocity. Our microscope projects the 2D Euclidean space to 2D hyperbolic space, then transforms it to a unit circle with the [Beltrami-Poincaré disc model](#). This unit circle is shown in the viewport of 600x600 resolution. Pressing the s,d,x,e keys, the virtual world can be translated to left, right, down and up with 0.1 unit. The timestep of the simulation is 0.01 sec even if the onIdle events come slower. Initially 2 particles exist, by each pressing of the SPACE, a new particle is introduced.

Simulation

$$\frac{d\vec{v}}{dt} = \frac{\sum \vec{F}}{m}$$

$$\frac{d\vec{r}}{dt} = \vec{v}$$

State: $\mathbf{r}_i, \mathbf{v}_i$

for($t = 0$; $t < T$; $t += dt$) { // onIdle

for each node i {

$$\sum \vec{F} = \sum \frac{q_i q_j}{2\pi\epsilon d_{ji}} \vec{e}_{ji} - \rho \vec{v}_i$$

$$\mathbf{v}_i += \sum \vec{F} / m \cdot dt$$

$$\mathbf{r}_i += \mathbf{v}_i \cdot dt$$

}

}

Steps of solution

- Atom: transformed circle (triangle-fan)
 - own or shared vao/vbo
 - Properties: mass, charge, (color)
 - State: position, velocity, force
- Virtual world: collection of atoms
- Simulation in onIdle
- Rendering in onDisplay
- World coordinates: Euclidean space
- Vertex shader:
 - Modeling transformation: (scaling +) translation
 - View transformation: translation
 - Projection transformation: hyperbolic/Poincaré
- Pixel shader: constant color



Graphics hardware and software

Program: Vasarely painting

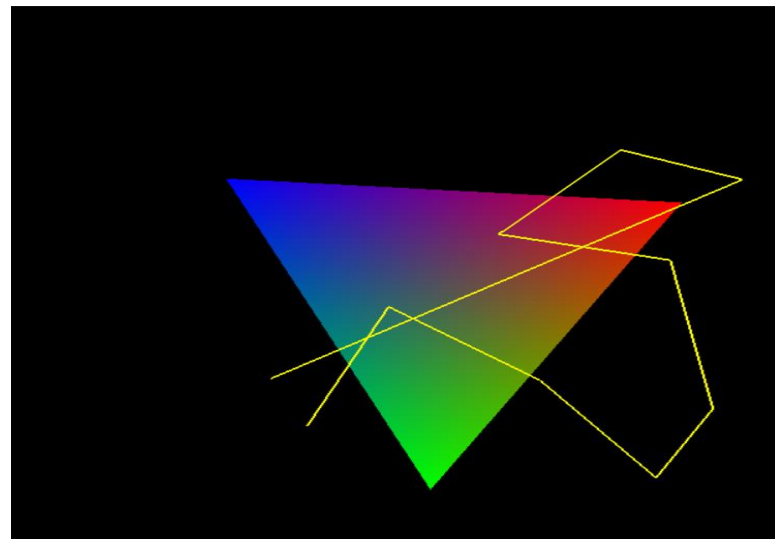
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Graphics hardware and software Program: Animation and interaction

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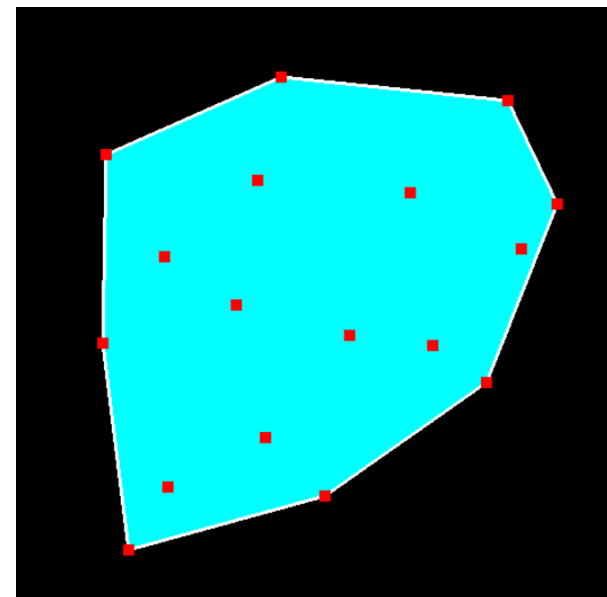




Graphics hardware and software

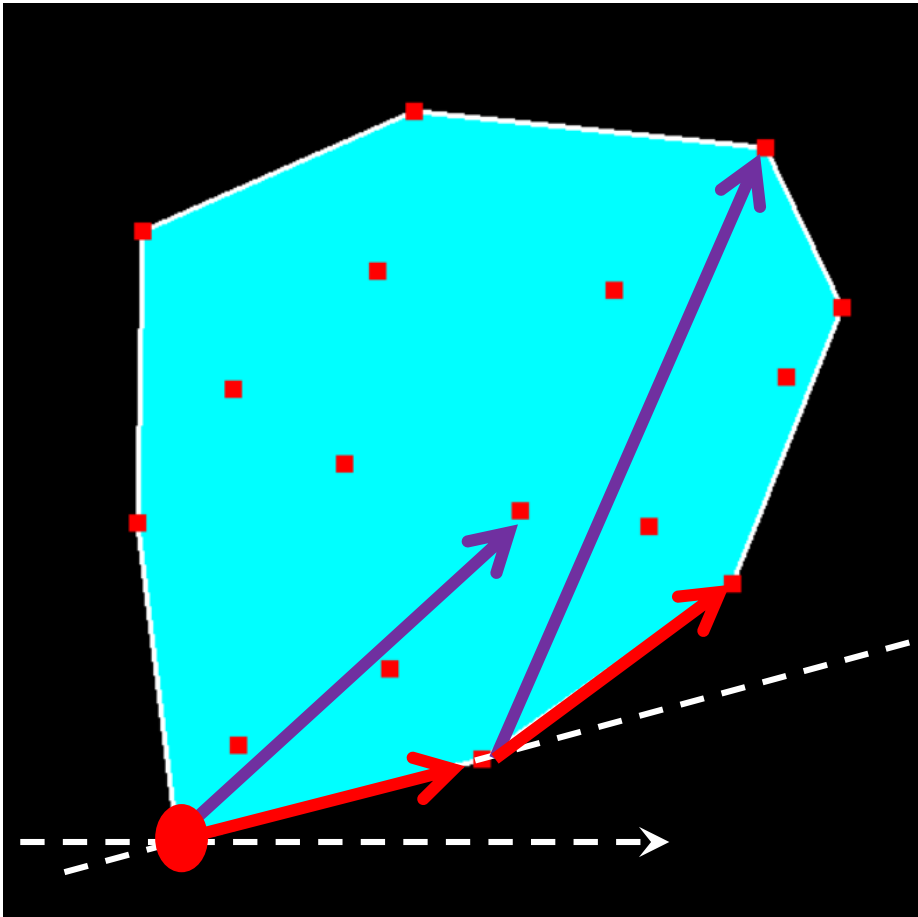
Program: Convex hull, interaction

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Convex hull

Minimal convex set that contains the given points



Start the lowest point, initial direction is to right

```
While (not back to start) {  
    Next point with minimal turn.  
}
```

Vertex and fragment shaders

Vertex shader:

```
layout(location = 0) in vec2 vertexPosition;  
  
void main() {  
    gl_Position = vec4(vertexPosition, 0, 1);  
}
```

Fragment shader:

```
uniform vec3 color;  
out vec4 fragmentColor;  
  
void main() {  
    fragmentColor = vec4(color, 1);  
}
```

Object

```
struct Object {
    unsigned int vao, vbo; // gpu
    std::vector<vec2> vtx; // cpu

    Object() {
        glGenVertexArrays(1, &vao); glBindVertexArray(vao);
        glGenBuffers(1, &vbo); glBindBuffer(GL_ARRAY_BUFFER, vbo);
        glEnableVertexAttribArray(0);
        glVertexAttribPointer(0, 2, GL_FLOAT, GL_FALSE, 0, NULL);
    }

    void updateGPU() {
        glBindVertexArray(vao); glBindBuffer(GL_ARRAY_BUFFER, vbo);
        glBufferData(GL_ARRAY_BUFFER, vtx.size() * sizeof(vec2),
                    &vtx[0], GL_DYNAMIC_DRAW);
    }

    void Draw(int type, vec3 color) {
        if (vertices.size() > 0) {
            glBindVertexArray(vao);
            gpuProgram.setUniform(color, "color");
            glDrawArrays(type, 0, vertices.size());
        }
    }
};
```

Convex hull

```
class ConvexHull {  
    Object p, h; // points and hull  
public:  
    void addPoint(vec2 pp) { p.vtx.push_back(pp); }  
    void update() {  
        if (p.vtx.size() >= 3) findHull();  
        p.updateGPU();  
        h.updateGPU();  
    }  
    vec2 * pickPoint(vec2 pp) {  
        for (auto& v : p.vtx) if (length(pp-v) < 0.05f) return &v;  
        return nullptr;  
    }  
void findHull();  
    void Draw() {  
        h.Draw(GL_TRIANGLE_FAN, vec3(0, 1, 1));  
        h.Draw(GL_LINE_LOOP, vec3(1, 1, 1));  
        p.Draw(GL_POINTS, vec3(1, 0, 0));  
    }  
};
```

Convex hull generation

```
void ConvexHull::findHull() {  
    h.vtx.clear();  
    vec2 * vStart = &p.vtx[0]; // Find lowest point  
    for (auto& v : p.vtx) if (v.y < vStart->y) vStart = &v;  
  
    vec2 vCur = *vStart, dir(1, 0), *vNext;  
    do { // find convex hull points one by one  
        float maxCos = -1;  
        for (auto& v : p.vtx) { // find minimal left turn  
            float len = length(v - vCur);  
            if (len > 0) {  
                float cosPhi = dot(dir, v - vCur) / len;  
                if (cosPhi > maxCos) { maxCos = cosPhi; vNext = &v; }  
            }  
        }  
        h.vtx.push_back(*vNext); // save as convex hull  
        dir = normalize(*vNext - vCur); // prepare for next  
        vCur = *vNext;  
    } while (vStart != vNext);  
}
```

Virtual world, initialization and display

```
ConvexHull * hull;
vec2 * pickedPoint = nullptr;

void onInitialization() {
    glViewport(0, 0, windowWidth, windowHeight);
    glLineWidth(2);
    glPointSize(10);
    hull = new ConvexHull;
    gpuProgram.create(vertexSrc, fragmentSrc, "fragmentColor");
}

void onDisplay() {
    glClearColor(0, 0, 0, 0);
    glClear(GL_COLOR_BUFFER_BIT);
    hull->Draw();
    glutSwapBuffers();
}
```

Controller

```
vec2 PixelToNDC(int pX, int pY) { // if full viewport
    return vec2(2.0f * pX / windowWidth - 1; // flip y axis
                1.0f - 2.0f * pY / windowHeight);
}

void onMouse(int button, int state, int pX, int pY) {
    if (button==GLUT_LEFT_BUTTON && state==GLUT_DOWN) {
        hull->addPoint(PixelToNDC(pX, pY));
        hull->update(); glutPostRedisplay(); // redraw
    }
    if (button==GLUT_RIGHT_BUTTON && state==GLUT_DOWN)
        pickedPoint = hull->pickPoint(PixelToNDC(pX, pY));
    if (button==GLUT_RIGHT_BUTTON && state==GLUT_UP)
        pickedPoint = nullptr;
}

void onMouseMotion(int pX, int pY) {
    if (pickedPoint) {
        *pickedPoint = vec2(PixelToNDC(pX, pY));
        hull->update(); glutPostRedisplay(); // redraw
    }
}
```

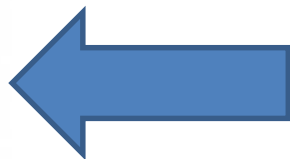
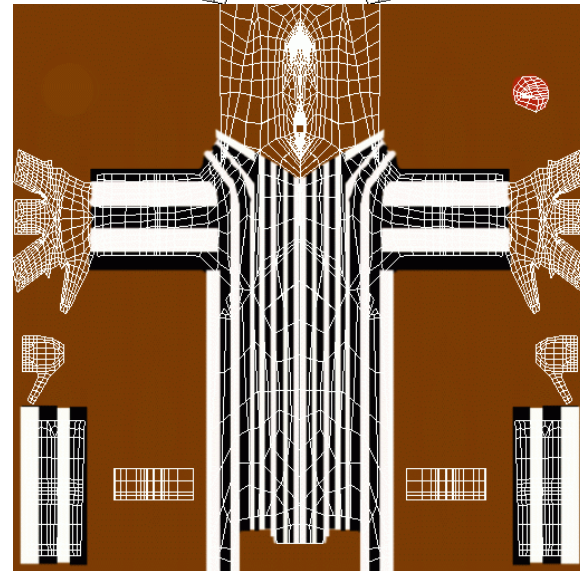
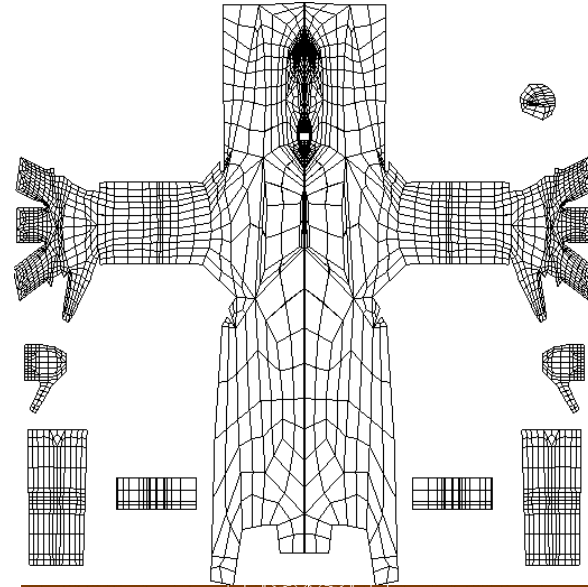
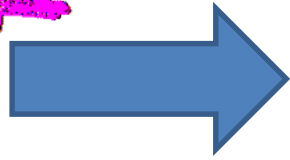
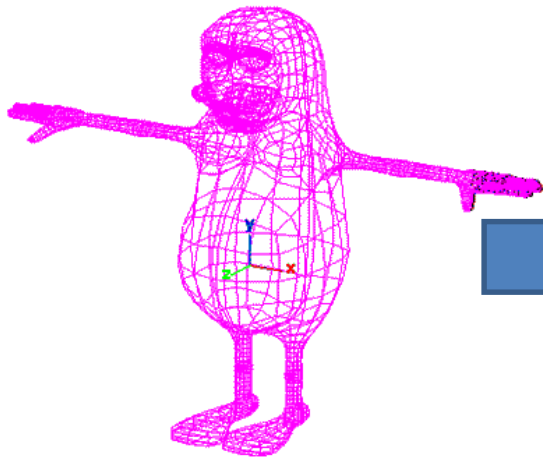

“Everything must be made as simple as possible. But not simpler.”
Albert Einstein

2D texturing

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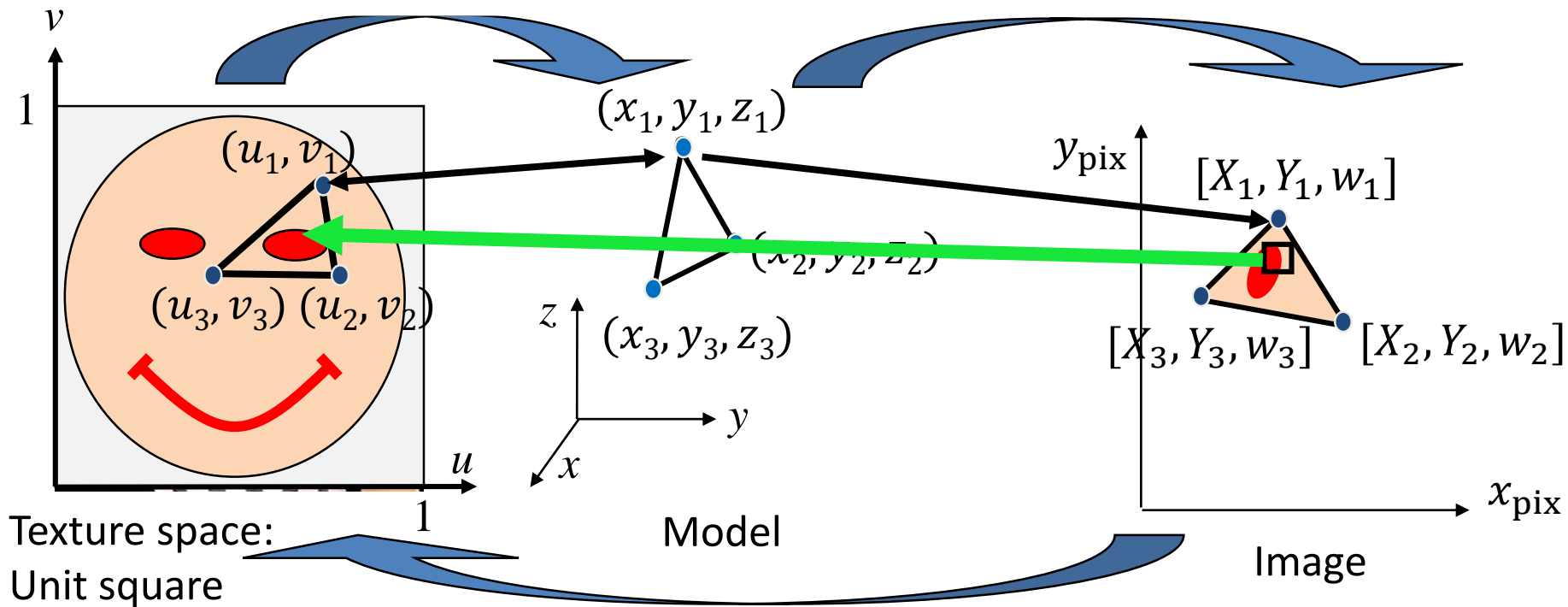


2D texturing



2D texturing

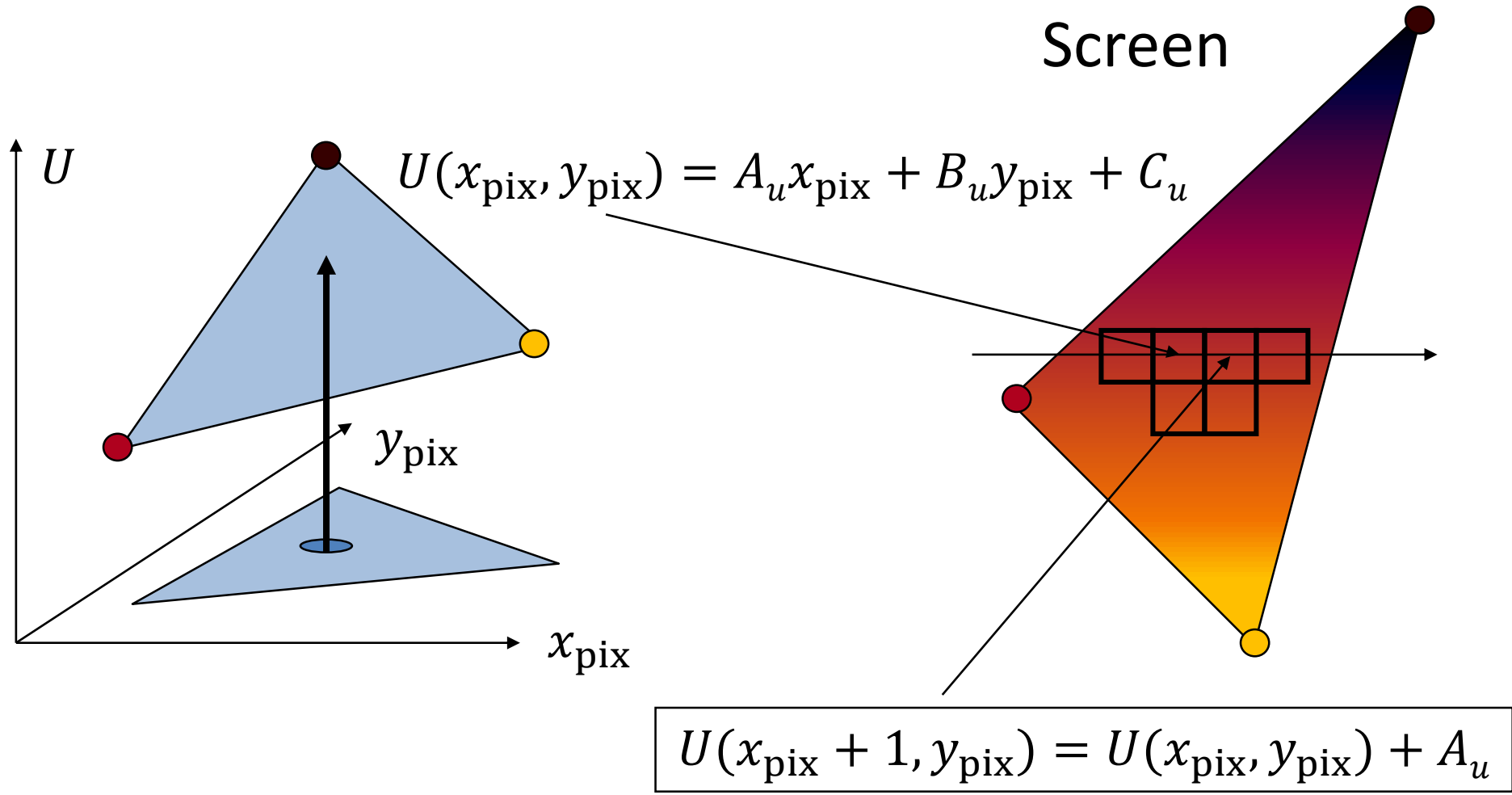
$$[X, Y, w] = [u, v, 1] \cdot \mathbf{P}$$
$$(x_{\text{pix}}, y_{\text{pix}}) = [X/w, Y/w]$$



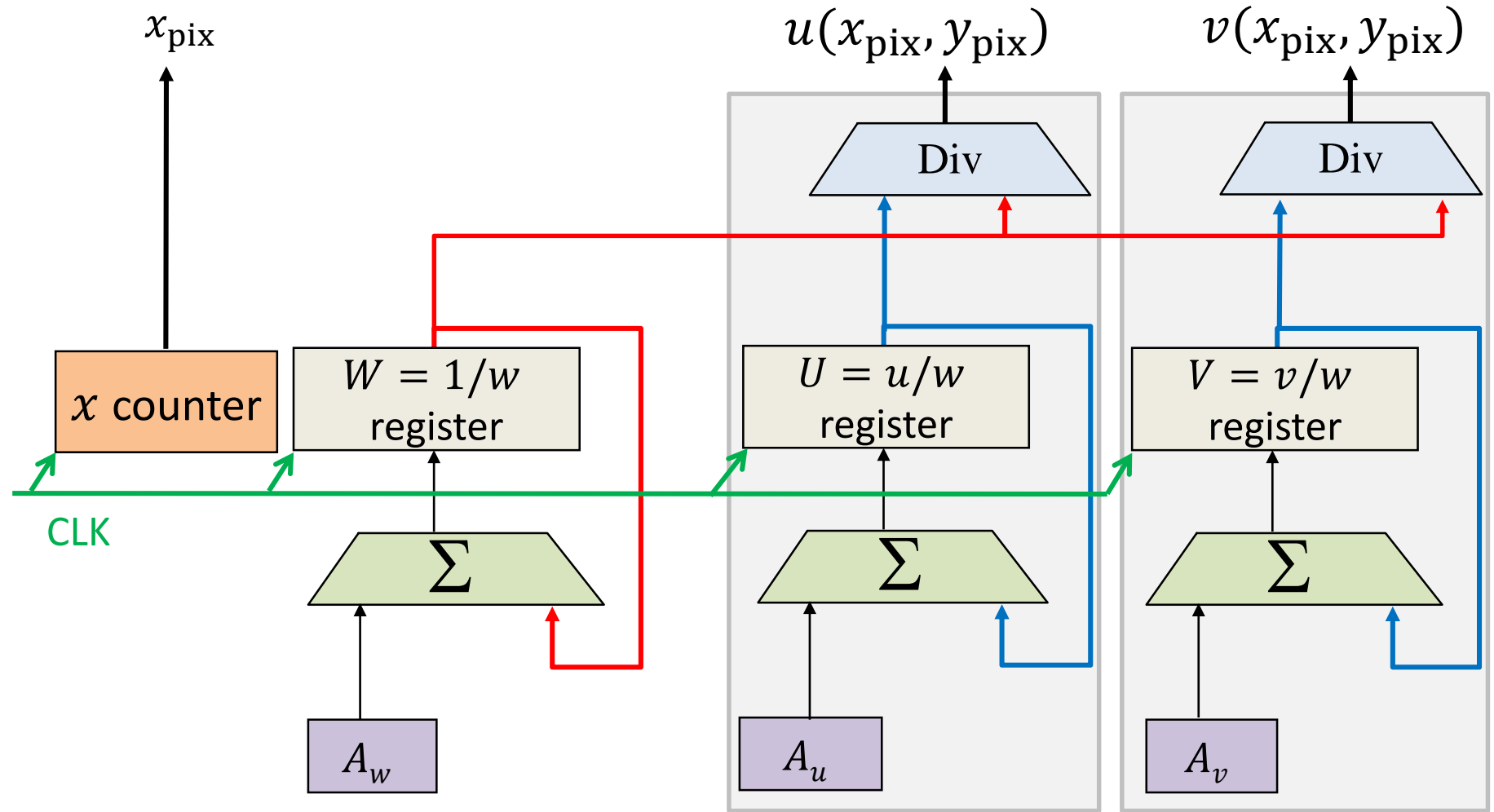
$$[u/w, v/w, 1/w] = [x_{\text{pix}}, y_{\text{pix}}, 1] \cdot \mathbf{P}^{-1}$$

$$[U, V, W] \rightarrow u = U/W, v = V/W$$

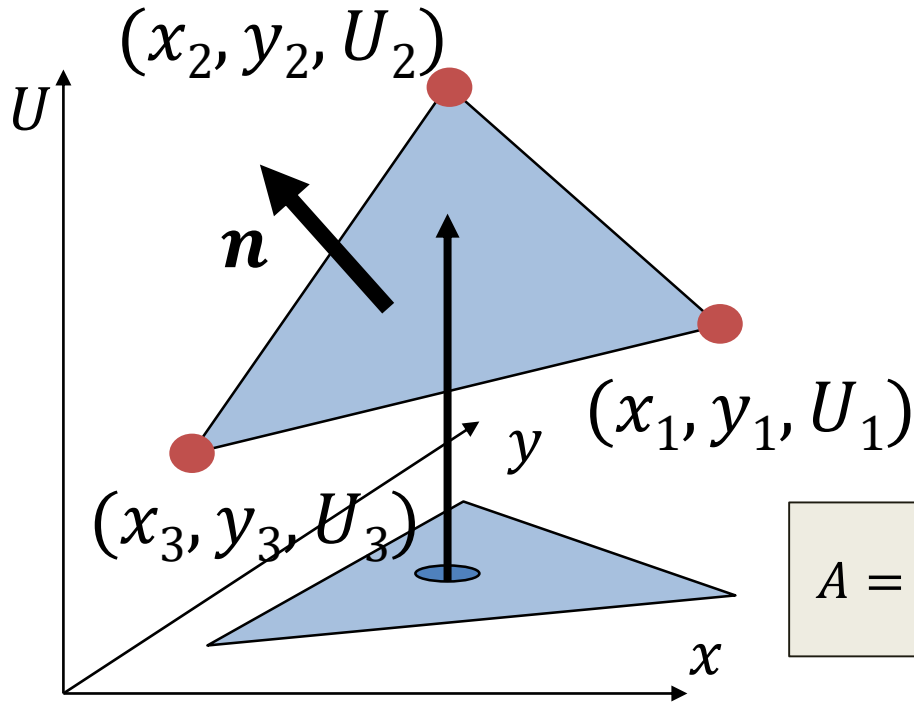
Interpolation



Interpolation hardware



Triangle setup



$$\begin{aligned} U_1 &= Ax_1 + By_1 + C \\ U_2 &= Ax_2 + By_2 + C \\ U_3 &= Ax_3 + By_3 + C \end{aligned}$$

$$A = \frac{(U_3 - U_1)(Y_2 - Y_1) - (Y_3 - Y_1)(U_2 - U_1)}{(X_3 - X_1)(Y_2 - Y_1) - (Y_3 - Y_1)(X_2 - X_1)}$$

$$\begin{aligned} U(x, y) &= Ax + By + C \\ n_x x + n_y y + n_u U + d &= 0 \end{aligned}$$

$$\mathbf{n} = (X_2 - X_1, Y_2 - Y_1, U_2 - U_1) \times (X_3 - X_1, Y_3 - Y_1, U_3 - U_1)$$

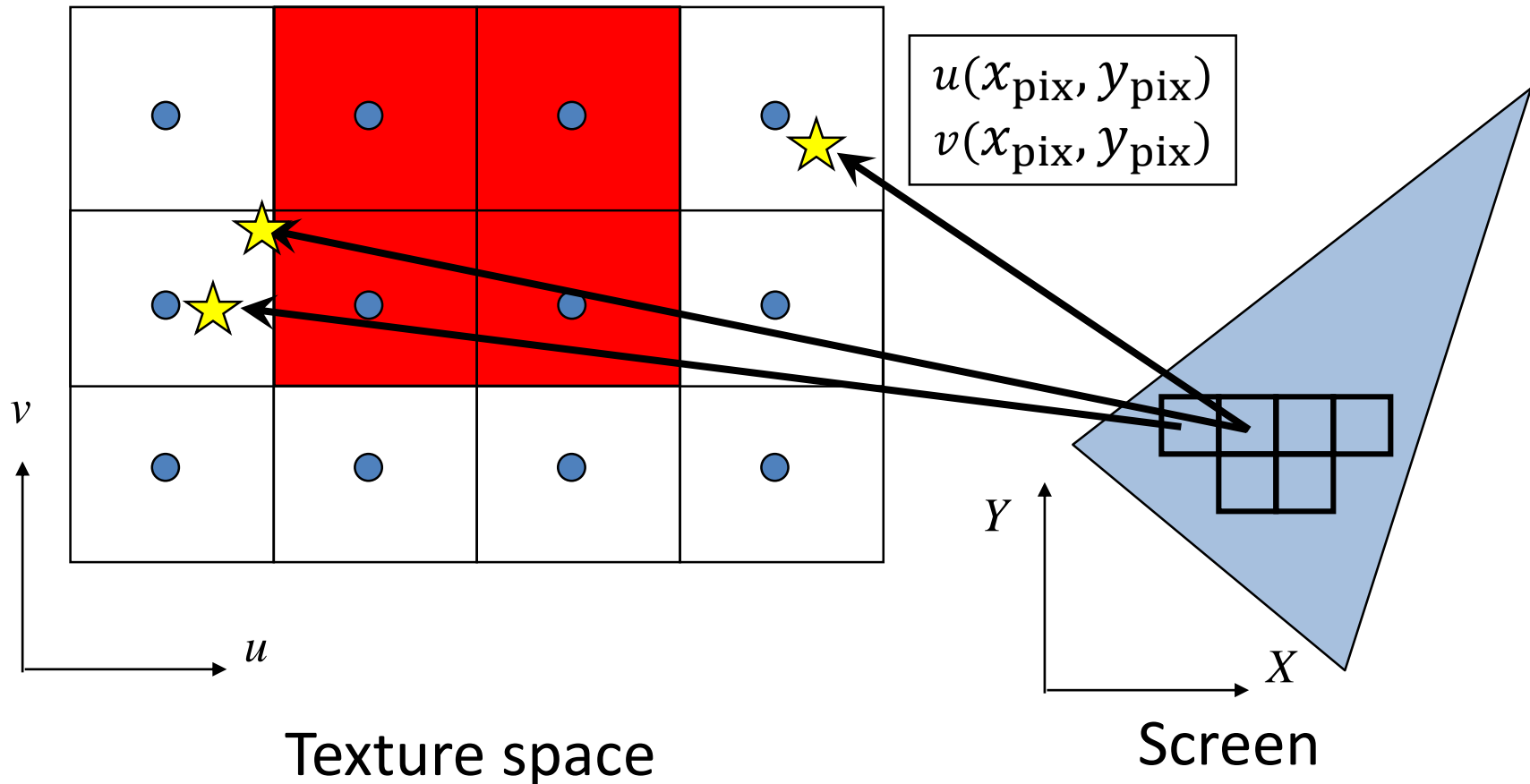
n_x

$-n_u$

Texture filtering (GL_NEAREST)

Magnification

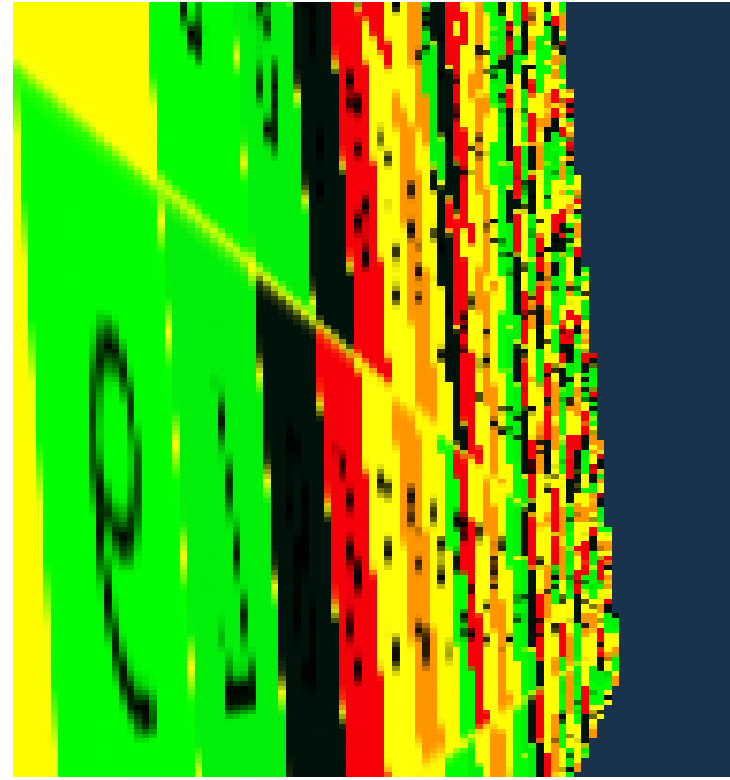
Minification



Correspondence of texture and image spaces



Magnification

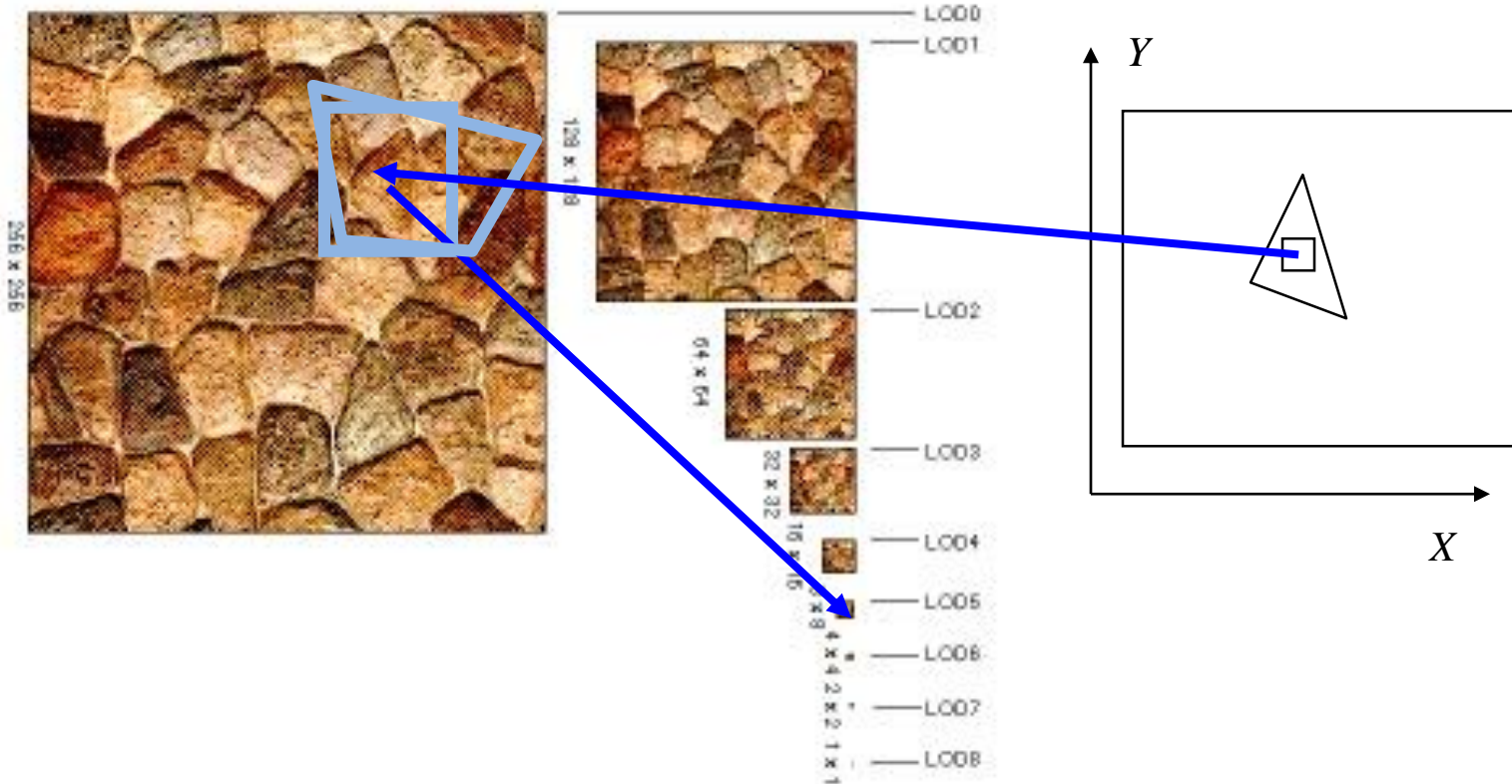


Minification

```
glTexParameteri(GL_TEXTURE_2D, GL_TEXTURE_MAG_FILTER, GL_NEAREST);  
glTexParameteri(GL_TEXTURE_2D, GL_TEXTURE_MIN_FILTER, GL_NEAREST);
```


Mip-map (multum in parvo)

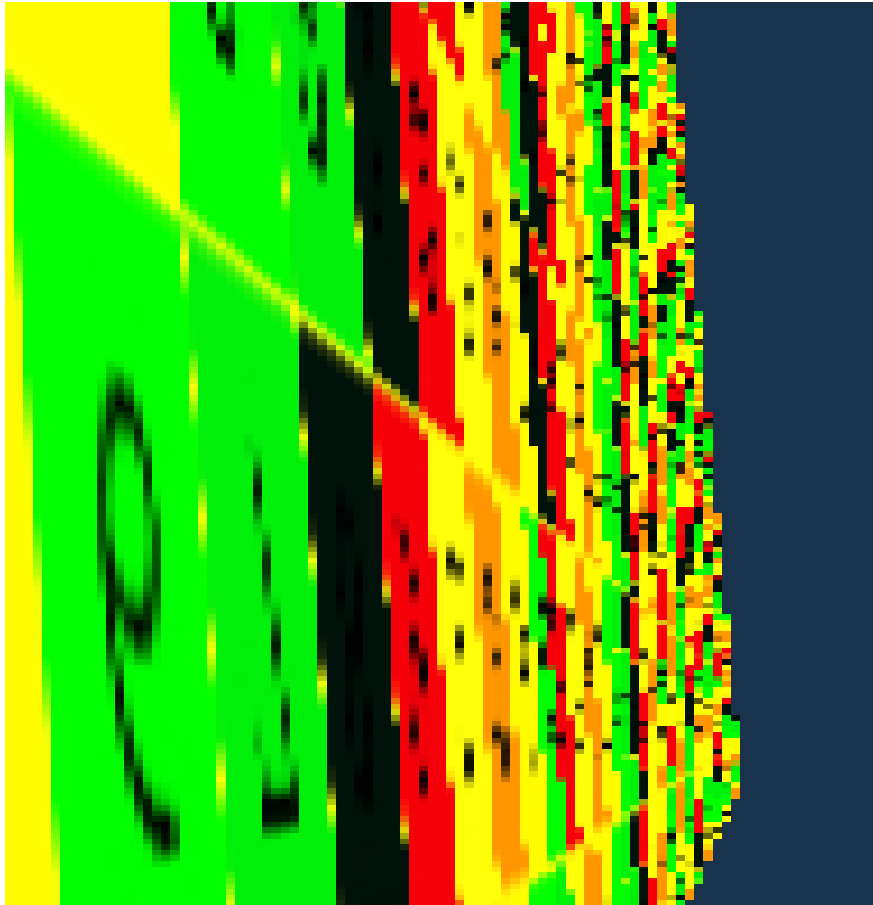
Good for minification



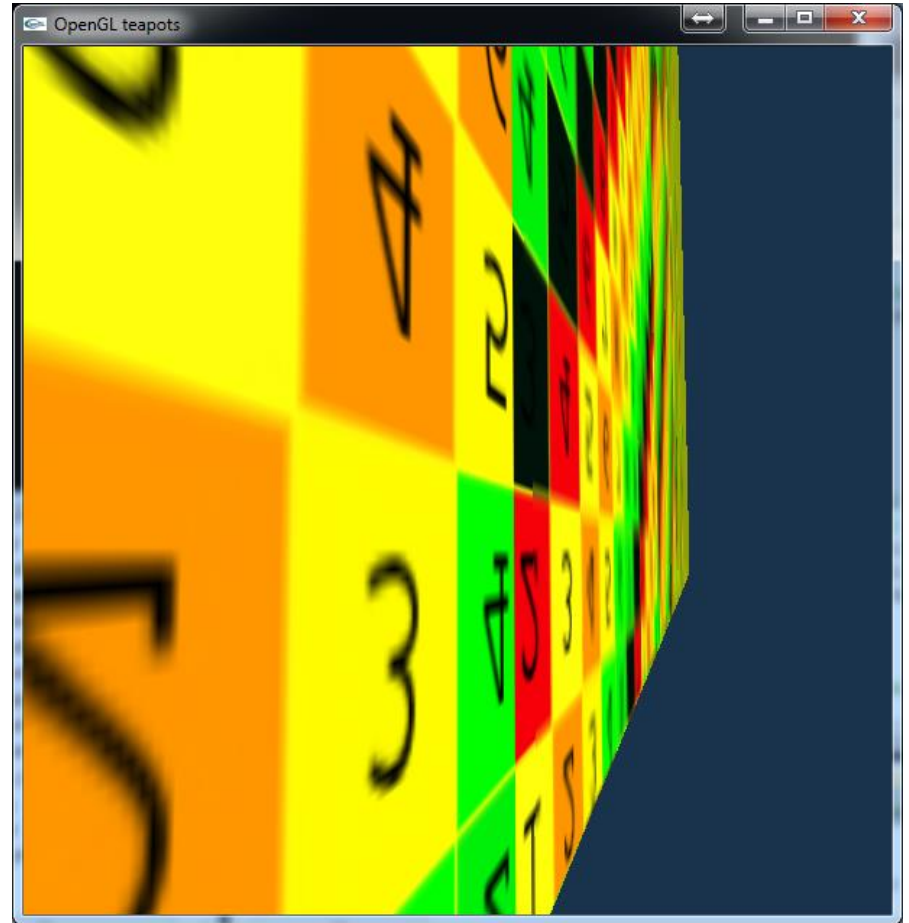
a) `glTexParameteri(GL_TEXTURE_2D, GL_TEXTURE_MIN_FILTER, GL_LINEAR_MIPMAP_NEAREST); // Mip-mapping`

b) `glTexParameteri(GL_TEXTURE_2D, GL_TEXTURE_MIN_FILTER, GL_LINEAR_MIPMAP_LINEAR); // Tri-linear filtering`

Mip-map (GL_LINEAR_MIPMAP_...)



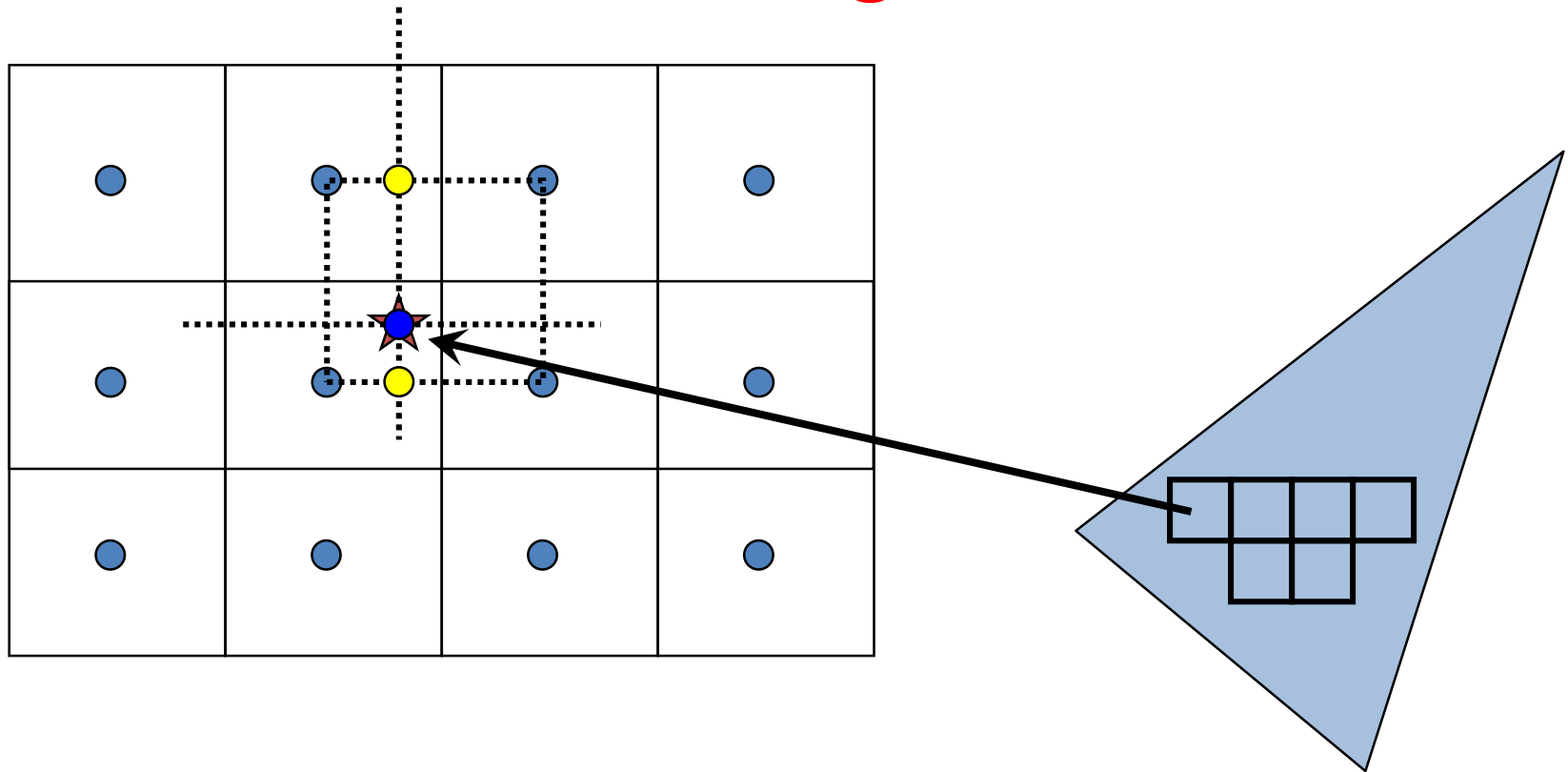
`GL_NEAREST`



`GL_LINEAR_MIPMAP_NEAREST`

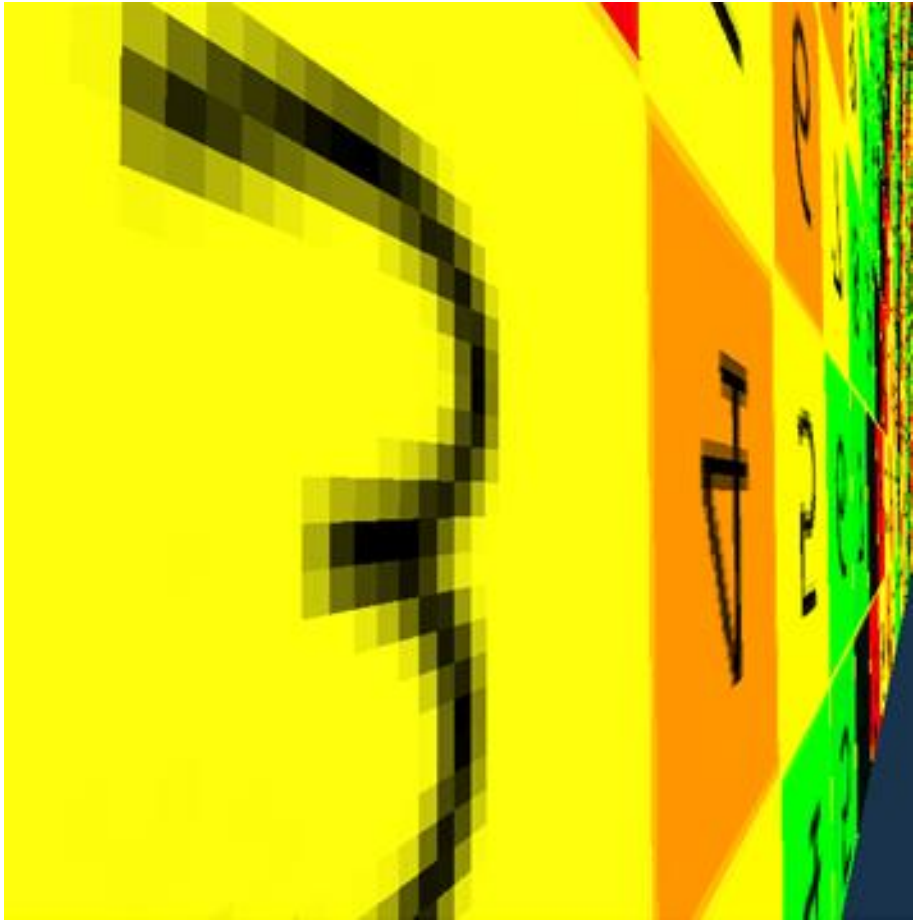
Bi-linear texture filtering (GL_LINEAR)

Good for magnification

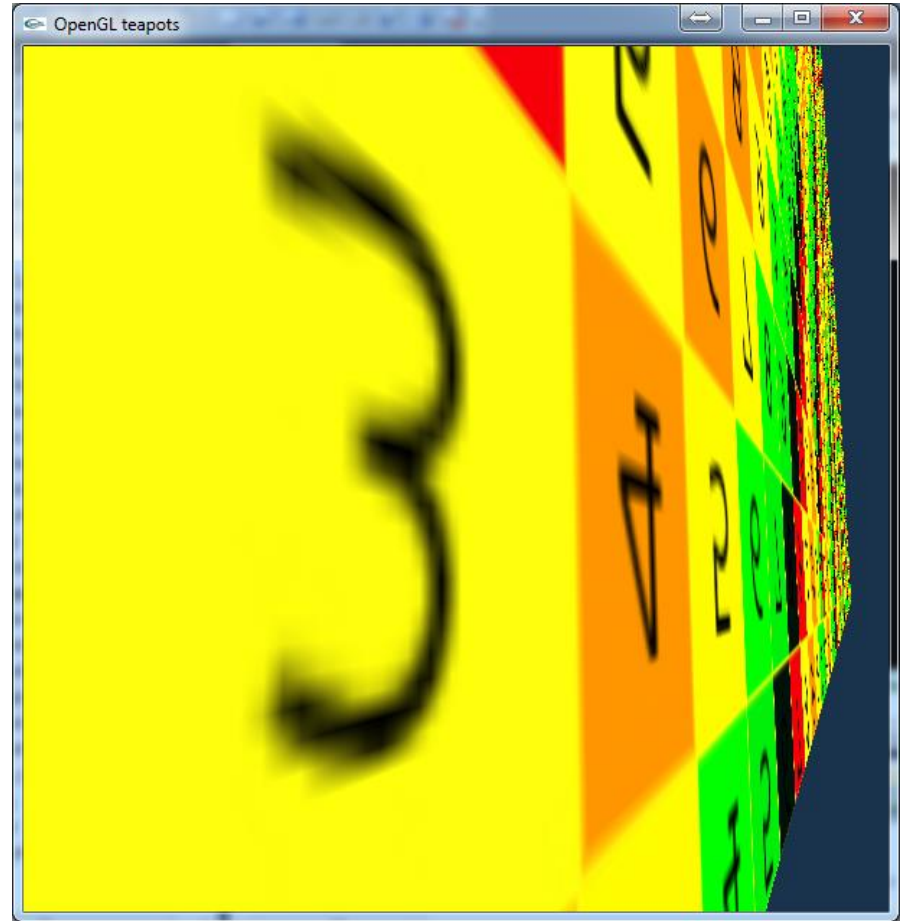


```
glTexParameteri (GL_TEXTURE_2D,  
                 GL_TEXTURE_MIN_FILTER, GL_LINEAR) ;  
glTexParameteri (GL_TEXTURE_2D,  
                 GL_TEXTURE_MAG_FILTER, GL_LINEAR) ;
```

Bi-linear filtering (GL_LINEAR)

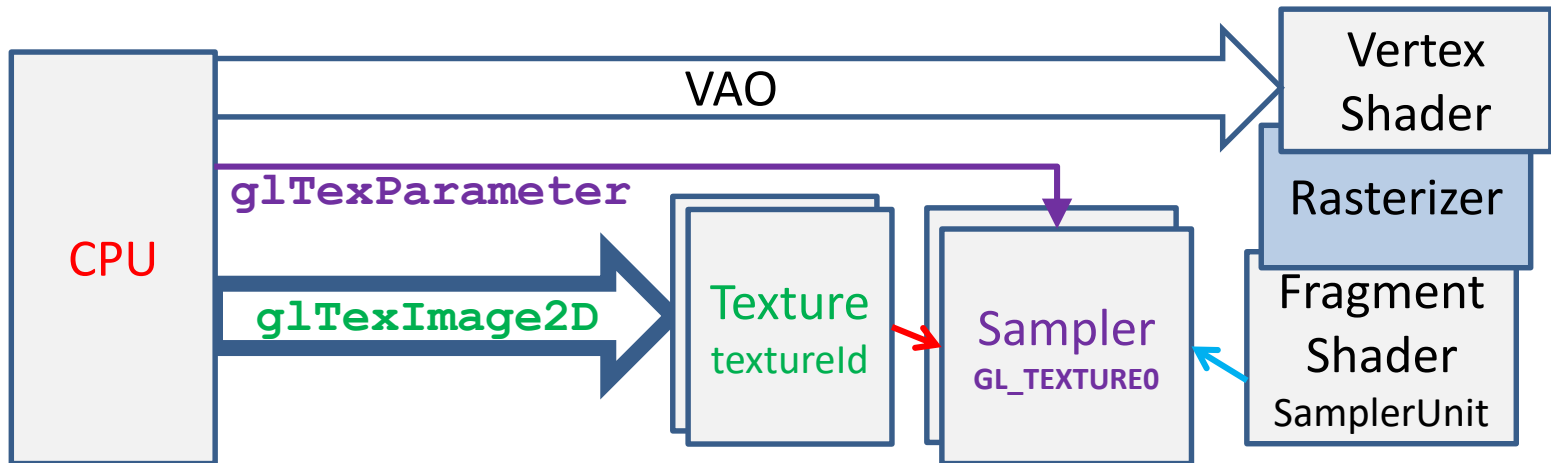


GL_NEAREST

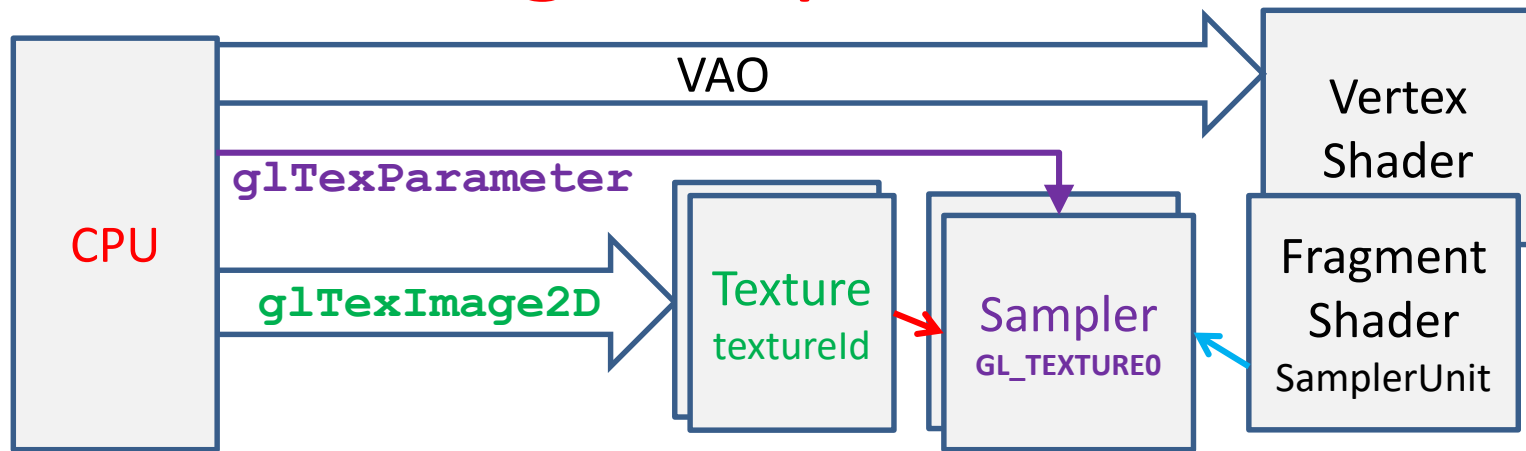


GL_LINEAR

Texturing support of the GPU



Texturing 1: Upload to GPU



```
unsigned int textureId;
```

Your responsibility

```
void UploadTexture(int width, int height, vector<vec4>& image) {
```

```
    glGenTextures(1, &textureId);
```

```
    glBindTexture(GL_TEXTURE_2D, textureId); // binding
```

Mip-map level

Border

```
    glTexImage2D(GL_TEXTURE_2D, 0, GL_RGBA, width, height, 0,
                 GL_RGBA, GL_FLOAT, &image[0]); //Texture -> GPU
```

```
    glTexParameteri(GL_TEXTURE_2D, GL_TEXTURE_MIN_FILTER, GL_NEAREST);
```

```
    glTexParameteri(GL_TEXTURE_2D, GL_TEXTURE_MAG_FILTER, GL_LINEAR);
```

```
}
```

Texturing 2: Equip objects with texture coordinates

```
glGenVertexArrays(1, &vao);
glBindVertexArray(vao);

glGenBuffers(2, vbo); // Generate 2 vertex buffer objects

// vertex coordinates: vbo[0] -> Attrib Array 0 -> vertices
glBindBuffer(GL_ARRAY_BUFFER, vbo[0]);
float vtxs[] = {x1, y1, x2, y2, ...};
glBufferData(GL_ARRAY_BUFFER, sizeof(vtxs), vtxs, GL_STATIC_DRAW);
glEnableVertexAttribArray(0);
glVertexAttribPointer(0, 2, GL_FLOAT, GL_FALSE, 0, NULL);

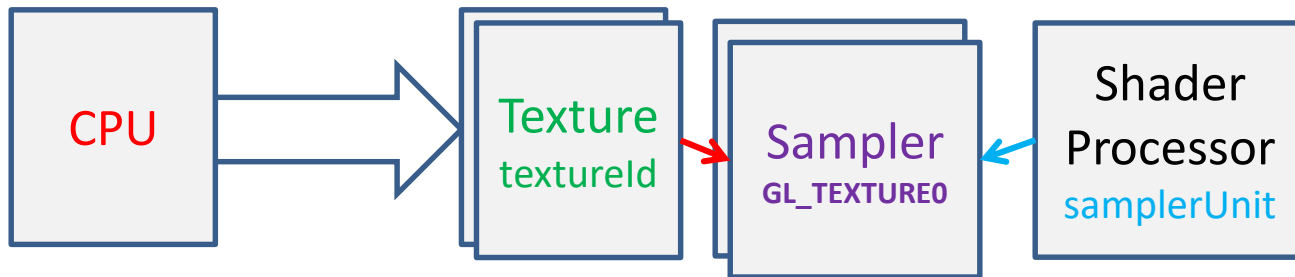
// vertex coordinates: vbo[1] -> Attrib Array 1 -> uvs
glBindBuffer(GL_ARRAY_BUFFER, vbo[1]);
float uvs[] = {u1, v1, u2, v2, ...};
glBufferData(GL_ARRAY_BUFFER, sizeof(uvs), uvs, GL_STATIC_DRAW);
glEnableVertexAttribArray(1);
glVertexAttribPointer(1, 2, GL_FLOAT, GL_FALSE, 0, NULL);
```

Texturing 3: Vertex és Pixel Shader

```
layout(location = 0) in vec2 vtxPos;  
layout(location = 1) in vec2 vtxUV;  
  
out vec2 texcoord;  
  
void main() {  
    gl_Position = vec4(vtxPos, 0, 1) * MVP;  
    texcoord = vtxUV;  
    ...  
}
```

```
uniform sampler2D samplerUnit;  
in vec2 texcoord;  
out vec4 fragmentColor;  
  
void main() {  
    fragmentColor = texture(samplerUnit, texcoord);  
}
```


Texturing 4: Active texture and sampler



```
unsigned int textureId;
```

```
void Draw( ) {
```

```
    int sampler = 0; // which sampler unit should be used
```

```
    int location = glGetUniformLocation(shaderProg, "samplerUnit");
```

```
    glUniform1i(location, sampler);
```

```
    glActiveTexture(GL_TEXTURE0 + sampler); // = GL_TEXTURE0
```

```
    glBindTexture(GL_TEXTURE_2D, textureId);
```

```
    glBindVertexArray(vao);
```

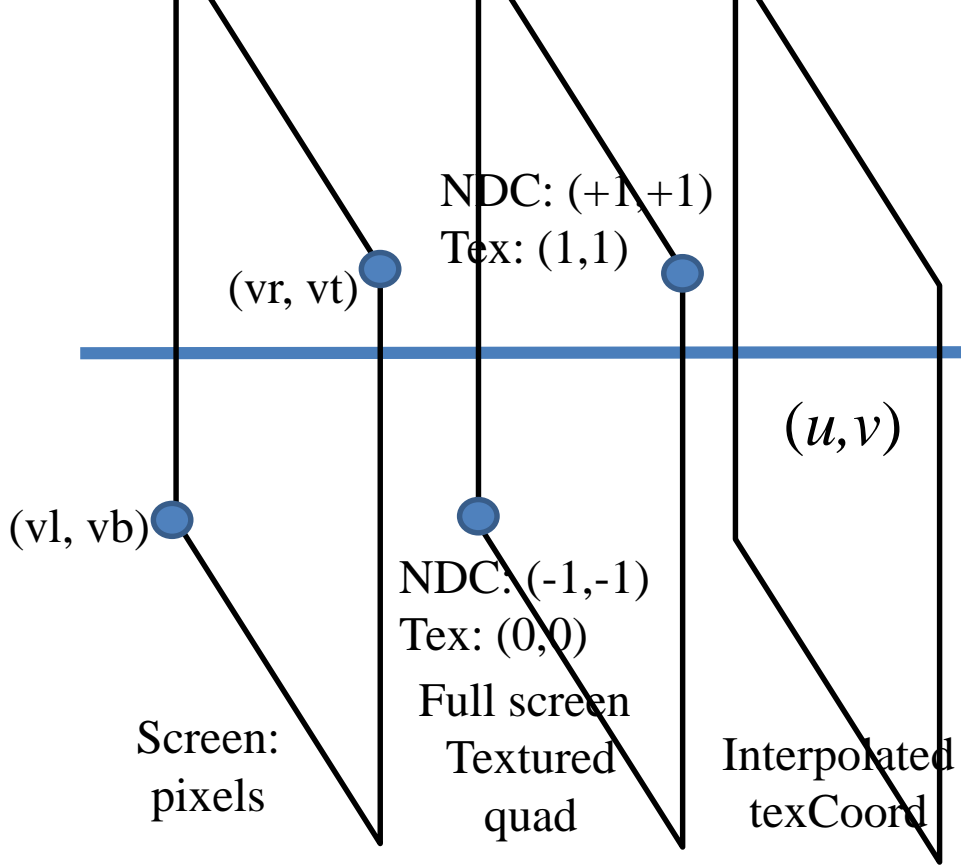
```
    glDrawArrays(GL_TRIANGLES, 0, nVtx);
```

```
}
```



```
float vtx = {-1,-1, 1,-1, 1,1, -1, 1};
glBufferData(GL_ARRAY_BUFFER,
             sizeof(vtx), vtx, GL_STATIC_DRAW);
```

Image viewer



```
uniform sampler2D textureUnit;
in vec2 uv;
out vec4 fragmentColor;

void main() {
    fragmentColor =
        texture(textureUnit, uv);
}
```

```
layout(location = 0) in vec2 vp; // Attrib Array 0
out vec2 uv; // output attribute

void main() {
    uv = (vp + vec2(1, 1)) / 2; // clipping to texture space
    gl_Position = vec4(vp.x, vp.y, 0, 1);
}
```

```
float vtx = {-1,-1, 1,-1, 1,1, -1, 1};
glBufferData(GL_ARRAY_BUFFER,
            sizeof(vtx), vtx, GL_STATIC_DRAW);
```

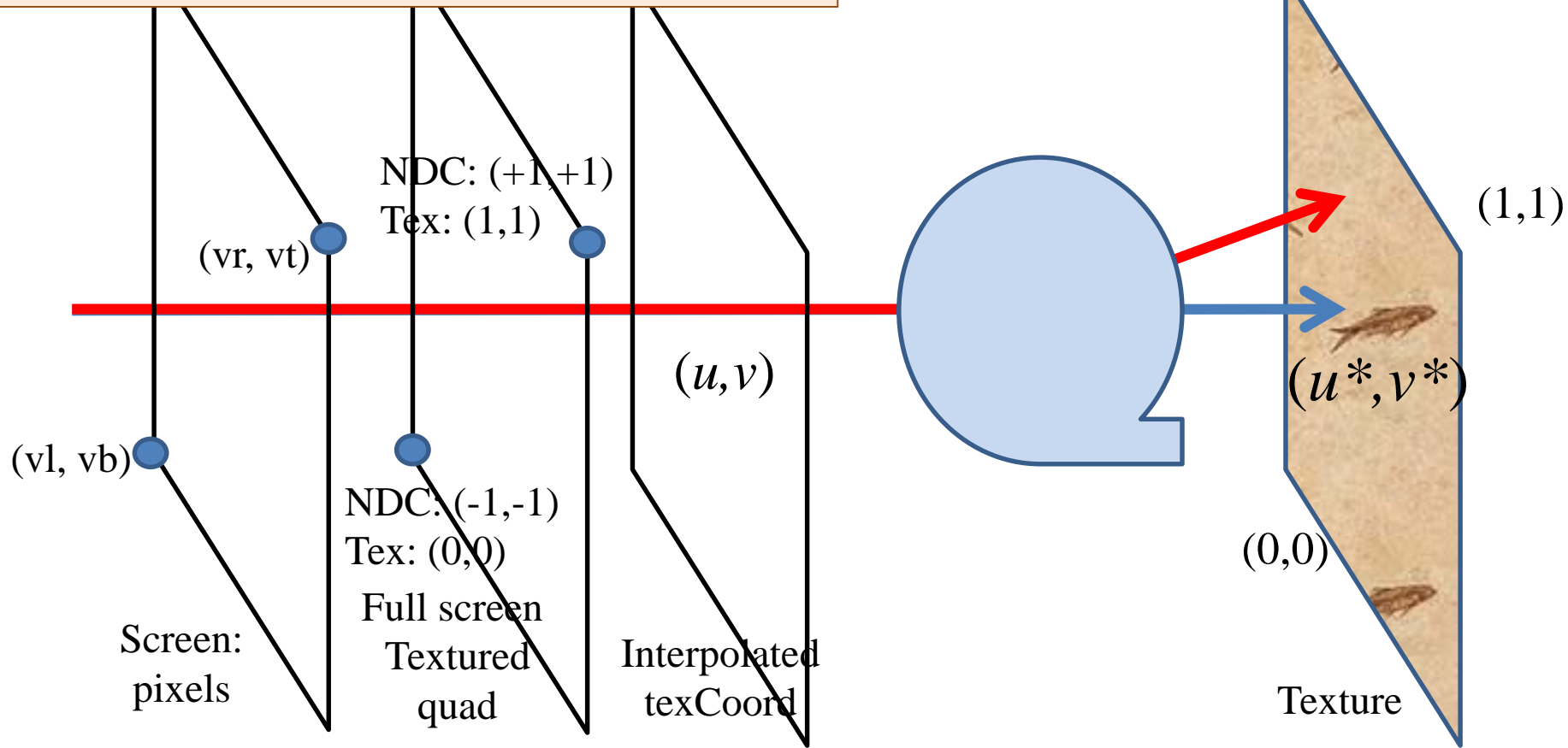
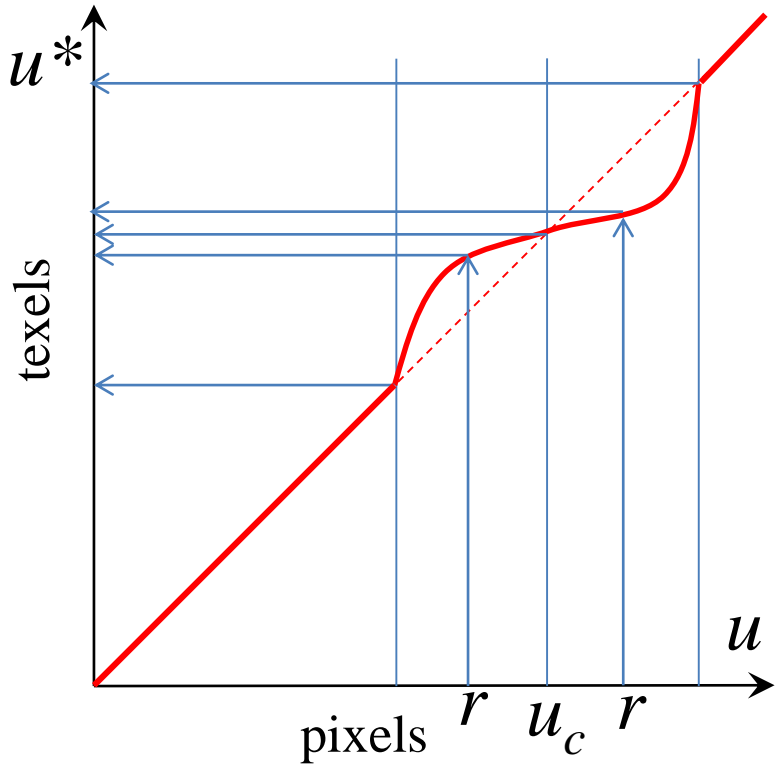


Image viewer

```
layout(location = 0) in vec2 vp; // Attrib Array 0
out vec2 uv; // output attribute

void main() {
    uv = (vp + vec2(1, 1)) / 2; // clipping to texture space
    gl_Position = vec4(vp.x, vp.y, 0, 1);
}
```



Magic lense



$$u^* = \frac{(u - u_c)^3}{r^2} + u_c \quad \text{if } |u - u_c| < r$$

```
uniform sampler2D textureUnit;
uniform vec2 uvc; // cursor position in texture space
in vec2 uv; // interpolated texture coordinates
out vec4 fragmentColor;

void main() {
    const float r2 = 0.05f;
    float d2 = dot(uv - uvc, uv - uvc);
    vec2 tuv = (d2 < r2) ? (uv - uvc) * d2 / r2 + uvc : uv;
    fragmentColor = texture(textureUnit, tuv);
}
```

Swirl



```
uniform sampler2D textureUnit;
uniform vec2 uvc; // cursor position in texture space

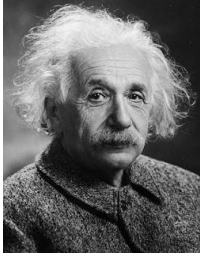
in vec2 uv;      // interpolated texture coordinates
out vec4 fragmentColor;

void main() {
    const float a = 8, alpha = 15;
    float ang = a * exp( -alpha * length(uv - uvc) );
    mat2 rotMat = mat2( cos(ang), sin(ang),
                       -sin(ang), cos(ang) );

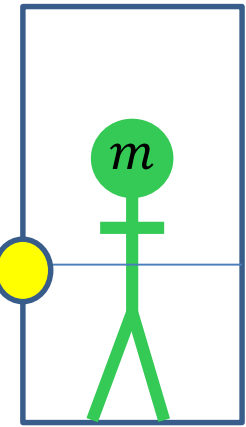
    vec2 tuv = (uv - uvc) * rotMat + uvc;
    fragmentColor = texture(textureUnit, tuv);
}
```

Gravity (black hole)

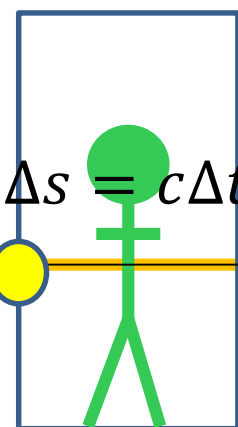




Equivalence principle



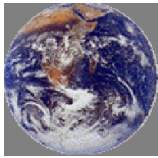
=



$$\Delta s = c\Delta t$$

$$\Delta d = \frac{g}{2} (\Delta t)^2 = \frac{\frac{fM}{2r^2} c^2}{c^2} (\Delta s)^2 = \frac{r_0}{4r^2} (\Delta s)^2$$

$\frac{r_0}{2}$: Schwarzschild radius



$$g = \frac{fM}{r^2}$$

g

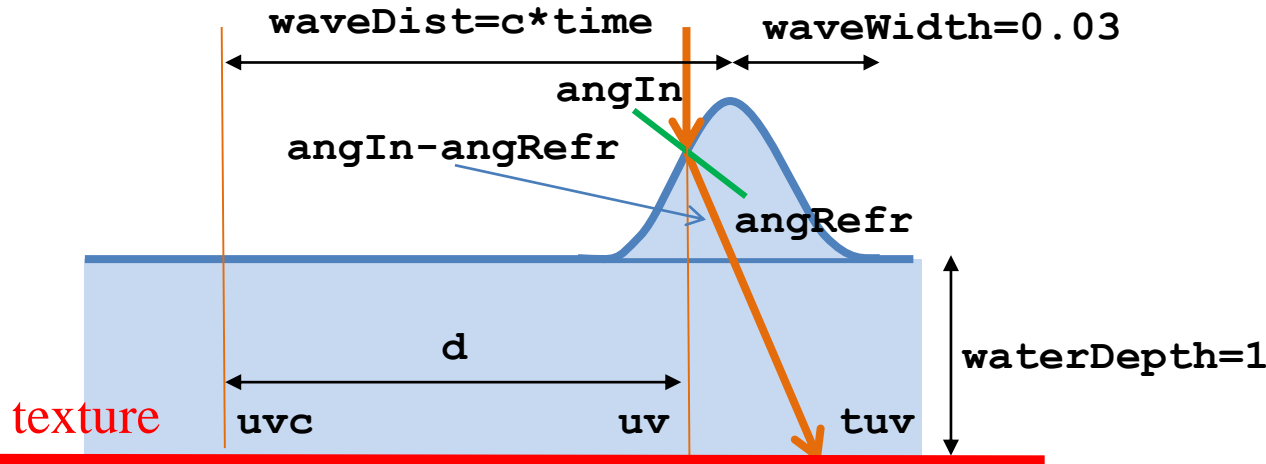
p

dir

Schwarzschild radius

```
void main() {
    const float r0 = 0.09f, ds = 0.001f;
    vec3 p = vec3(uv,0), dir = vec3(0,0,1), blackhole = vec3(uvc,0.5f);
    float r2 = dot(blackhole - p, blackhole - p);
    while (p.z < 1 && r2 > r0 * r0) {
        p += dir * ds;
        r2 = dot(blackhole - p, blackhole - p);
        vec3 gDir = (blackhole - p)/sqrt(r2); // gravity direction
        dir = normalize(dir * ds + gDir * r0 / r2 / 4 * ds * ds);
    }
    if (p.z >= 1) fragmentColor = texture(textureUnit,vec2(p.x,p.y));
    else fragmentColor = vec4(0, 0, 0, 1);
}
```

Wave



```
uniform float time;
const float PI = 3.14159265, n = 1.33, c = 0.1, aMax = 0.1;

void main() {
    float d = length(uv - uvc), waveDist = c * time;
    if (abs(d - waveDist) < waveWidth) {
        float angIn = aMax/waveDist * sin((waveDist-d)/waveWidth*PI);
        float angRefr = asin(sin(angIn)/n);
        vec2 dir = (uv - uvc)/d;
        vec2 tuv = uv + dir * tan(angIn - angRefr) * waterDepth;
        fragmentColor = texture(textureUnit, tuv);
    } else {
        fragmentColor = texture(textureUnit, uv);
    }
}
```