A Microfacet Based Coupled Specular-Matte BRDF Model with Importance Sampling

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Abstract

This paper presents a BRDF model based on the analysis of the photon collisions with the microfacets of the surface. The new model is not only physically plausible, i.e. symmetric and energy conserving, but provides other important features of real materials, including the off-specular peak and the mirroring limit case. Using theoretical considerations the reflected light is broken down to a specular component representing single reflections and a matte component accounting for multiple reflections and re-emissions of previously absorbed photons. Unlike most of the previous models, the proportion of the matte and specular components is not constant but varies with the viewing angle. In order to keep the resulting formulae simple, several approximations are made, which are quite accurate but allow for tabulation, fast calculation and even for accurate importance sampling.

1. Introduction

In order to render realistic images, we have to use realistic material models. Material models are usually defined by Bi-directional Reflectance Distribution Functions (BRDFs) that describe the chance of reflection for different pairs of incoming and outgoing light directions. When introducing BRDF models, the following notations are used: $\vec{N}$ is the unit normal vector of the surface, $\vec{L}$ is a unit vector pointing towards the light source, $\vec{V}$ is a unit vector pointing towards the camera, $\vec{H}$ is a unit vector that is halfway between $\vec{L}$ and $\vec{V}$, θ' is the angle between $\vec{L}$ and $\vec{N}$, θ is the angle between $\vec{V}$ and $\vec{N}$, α is the angle between $\vec{H}$ and $\vec{N}$ and β is the angle between $\vec{V}$ and $\vec{H}$ and similarly between $\vec{L}$ and $\vec{H}$. Angles θ' and θ are also called the incoming and outgoing angles, respectively.

A BRDF can be derived from the probability density function of the reflection. Suppose that a photon comes from direction $\vec{L}$. The reflection density $w(\vec{L},\vec{V})$ describes the probability that the photon leaves the surface at a differential solid angle around $\vec{V}$ given it comes from $\vec{L}$:

$$w(\vec{L},\vec{V})\cdot d\omega_{\vec{V}} =$$

$$\Pr\{\text{photon is reflected to } d\omega_{\vec{V}} \text{ around } \vec{V} \mid \text{coming from } \vec{L}\}.$$

The BRDF is the reflection density divided by the cosine of the outgoing angle:

$$f_r(\vec{L},\vec{V}) = \frac{w(\vec{L},\vec{V})}{\cos\theta}.$$

A realistic BRDF model is expected not to violate physics laws including the *Helmholtz-symmetry*, and the law of *energy conservation*, and to mimic the features of real materials.

The Helmholtz principle states that light and view directions can be exchanged in the BRDF, i.e. $f_r(\vec{L},\vec{V}) = f_r(\vec{V},\vec{L})$. According to energy conservation, a surface — provided that it is not a light source — cannot reflect more photons (more power) to the complete hemisphere Ω than what have been received, thus the *albedo* defined by

$$a(\vec{L}) = \Pr\{\text{photon is reflected} \mid \text{coming from } \vec{L}\} =$$

$$\int_{\Omega} w(\vec{L},\vec{V})\, d\omega_{\vec{V}} = \int_{\Omega} f_r(\vec{L},\vec{V})\cdot\cos\theta\, d\omega_{\vec{V}}$$

cannot be greater than 1. For *isotropic model*s, reflections of illuminations from incident directions with the same θ' have rotational symmetry, thus the albedo of a given point depends only on incident angle θ'. To emphasize this, the

Figure 1: *Photos of a specular table from different angles. Note that the table becomes more mirror-like for greater viewing angles*

albedo will be denoted by $a(\theta')$. Due to the symmetry of the BRDF, the albedo can also be presented in the following form:

$$a(\vec{V}) = \int_{\Omega'} f_r(\vec{L},\vec{V}) \cdot \cos\theta' \, d\omega_{\vec{L}}.$$

Note that this is the integral of the rendering equation if the incoming radiance is 1, thus the albedo can also be interpreted as the response to homogeneous unit illumination [16].

Concerning other important features of real materials, the following properties are worth mentioning. If the surface is perfectly smooth, then the reflection is described by the Fresnel function that is based on the refraction index and the extinction coefficient. At grazing angles glossy materials become quite good mirrors, that is, specularity increases while matte behavior fades (figure 1). Interestingly, the maximum of the reflectance is not at the ideal reflection direction at grazing angles, but is lower than that. This phenomenon is called the *off-specular peak.*

There are two uses of BRDFs in rendering. On the one hand, when the illumination of a point is known, then the reflected radiance can be obtained by multiplying the indicent irradiance by the BRDF to obtain the reflected radiance. On the other hand, random-walk algorithms generate random light-paths to find the illumination coming from multiple reflections. When doing so, a random direction should be found in a way that the variance of the random estimation is small. The variance can be reduced by importance sampling which requires the probability density of random directions to mimic $f_r \cdot \cos\theta$ for shooting (i.e. when the light-path is followed from the light-source towards the eye) and $f_r \cdot \cos\theta'$ for gathering (i.e. when the light-path is followed from the eye towards the light-source).

BRDF models can be elaborated in two different ways. They can be simple mathematical constructions, when the validity is determined by checking the symmetry and the energy conservation. These models are called physically plausible *empirical models*[11, 12, 6]. On the other hand, BRDFs can also be constructed by the careful analysis of the light-surface interaction, which results in the so called *physically based* models[17, 4, 7] . Physically based models are more natural, but are usually quite complex. Empirical models are simpler and some of them are also good for importance sampling, but may have artificial features.

In this paper a BRDF model is presented that meets all the requirements of physical plausibility and all the other mentioned features. To derive the model, physically based analysis is carried out, which is similar to that presented by Cook and Torrance and by Shirley et. al.[14]. However, the results are generalized and simplified with the aim of allowing fast computations and importance sampling.

2. The model of microfacet reflection

Reflection can be rigorously analyzed by modeling the surface irregularities by probability distributions, as has been proposed by Torrance, Sparrow[17], Cook[4] and Blinn. In their model, the surface is assumed to consist of randomly oriented perfect mirrors, so-called *microfacets*. This model was also generalized to more accurately mimic the scattering of electromagnetic waves by He et. al. in [7].

A photon may be absorbed or reflected when colliding with a microfacet. Reflected photons may leave the surface or may meet other microfacets, while absorbed photons can be re-emitted into a direction that is independent of the incident direction. The single reflection of photons is responsible for the specular effects and is relatively easy to analyze. In this paper a physically based formula is proposed for this *specular component*. The other component is called the *matte component*, which includes the multiple scattering and re-emission, and is very difficult to describe analytically. This component is usually handled as a constant diffuse factor, but this is against practical observations that show that the ratio of the powers of the matte and specular reflections is not constant but specular reflections become dominant at grazing angles. This type of interdependence was first discussed in [14] where a coupling was established with the ideal mirror component. In this paper a simple and physically plausible construction is proposed for the matte component that is coupled with the glossy specular reflection. In the next sections the derivation of the specular component is presented first, then the matte component is defined to represent the multiple reflections and re-emissions which are missing from the specular part. Thus the specular and matte part are not independent but are coupled depending on the viewing angle.

2.1. Specular component

This section discusses the derivation of the BRDF $f_{r,spec}(\vec{L},\vec{V})$ for the specular part. Our specular BRDF is a simplification of the Cook-Torrance model. For the sake of completeness, a pobabilistic derivation of the original Cook-Torrance model is presented in the appendix. This also allows us to use the partial results of the development.

The Cook-Torrance model proposes the following BRDF supposing $P_{\vec{H}}(\vec{H})$ to be the Beckmann distribution:

$$f_r(\vec{L},\vec{V}) = \frac{P_{\vec{H}}(\vec{H})}{4(\vec{N}\cdot\vec{L})(\vec{N}\cdot\vec{V})} \cdot G(\vec{N},\vec{L},\vec{V}) \cdot F(\lambda,\vec{H}\cdot\vec{L}), \quad (1)$$

where $P_{\vec{H}}$ is the probability density of halfway vector $\vec{H}$, F is the Fresnel function and

$$G(\vec{N},\vec{L},\vec{V}) = \min\{2\cdot\frac{(\vec{N}\cdot\vec{H})\cdot(\vec{N}\cdot\vec{V})}{(\vec{V}\cdot\vec{H})}, 2\cdot\frac{(\vec{N}\cdot\vec{H})\cdot(\vec{N}\cdot\vec{L})}{(\vec{L}\cdot\vec{H})}, 1\}$$

is the geometric factor (see appendix for details).

2.1.1. Simplification of the geometric term

Analyzing formula (1), we can note that it is rather complicated and can be numerically unstable since the $(\vec{N}\cdot\vec{L})$ and $(\vec{N}\cdot\vec{V})$ factors can be arbitrarily small. Note also that it is not appropriate for importance sampling since it is quite difficult to find a probability density that is proportional to the cosine weighted BRDF. On the other hand, the minimum in the geometric term makes the BRDF not differentiable, which is against practical observations[15]. In order to overcome these problems, the formula is simplified. Let us consider the geometric term divided by the $2(\vec{N}\cdot\vec{L})\cdot(\vec{N}\cdot\vec{V})$ product:

$$\frac{G(\vec{N},\vec{L},\vec{V})}{2(\vec{N}\cdot\vec{L})(\vec{N}\cdot\vec{V})} =$$

$$\min\left\{\frac{(\vec{N}\cdot\vec{H})}{(\vec{V}\cdot\vec{H})(\vec{N}\cdot\vec{L})}, \frac{(\vec{N}\cdot\vec{H})}{(\vec{L}\cdot\vec{H})(\vec{N}\cdot\vec{V})}, \frac{1}{2(\vec{N}\cdot\vec{V})(\vec{N}\cdot\vec{L})}\right\}.$$

Instead of computing all the three terms and taking their minimum, a common term is found in all members that is a good approximation of this minimum. The first term can be expressed as:

$$\frac{(\vec{N}\cdot\vec{H})}{(\vec{V}\cdot\vec{H})(\vec{N}\cdot\vec{L})} = \frac{1}{1+(\vec{L}\cdot\vec{V})} + \frac{\vec{N}\cdot\vec{V}}{(\vec{N}\cdot\vec{L})(1+(\vec{L}\cdot\vec{V}))}.$$

Similarly, the second term is:

$$\frac{(\vec{N}\cdot\vec{H})}{(\vec{L}\cdot\vec{H})(\vec{N}\cdot\vec{V})} = \frac{1}{1+(\vec{L}\cdot\vec{V})} + \frac{\vec{N}\cdot\vec{L}}{(\vec{N}\cdot\vec{V})(1+(\vec{L}\cdot\vec{V}))}.$$

Note that when selecting the minimum of the two terms, then

$$\min\left\{\frac{(\vec{N}\cdot\vec{H})}{(\vec{V}\cdot\vec{H})(\vec{N}\cdot\vec{L})}, \frac{(\vec{N}\cdot\vec{H})}{(\vec{L}\cdot\vec{H})(\vec{N}\cdot\vec{V})}\right\} \approx \frac{1}{1+(\vec{L}\cdot\vec{V})} \quad (2)$$

seems to be a fairly good approximation. On the other hand, it also meets the requirement that it is always smaller than the third member $1/2(\vec{N}\cdot\vec{V})(\vec{N}\cdot\vec{L})$ that is also included in the geometric term. To demonstrate this, it is enough to show that

$$1+(\vec{L}\cdot\vec{V}) \geq 2(\vec{N}\cdot\vec{V})(\vec{N}\cdot\vec{L}).$$

Let us denote the angle of $\vec{N}$ and $\vec{L}$ by θ' and the angle of $\vec{N}$ and $\vec{V}$ by θ. The right hand side of this expression is

$$2(\vec{N}\cdot\vec{V})(\vec{N}\cdot\vec{L}) = 2\cos\theta\cdot\cos\theta' = \cos(\theta+\theta') + \cos(\theta-\theta').$$

If vectors $\vec{L}$, $\vec{V}$ and $\vec{N}$ are in the same plane, the angle of $\vec{L}$ and $\vec{V}$ is either $\theta+\theta'$ or $\theta-\theta'$, thus the left hand side cannot be less than the right hand side, which proves our statement. If the vectors are not coplanar, the angle of $\vec{L}$ and $\vec{V}$ gets smaller, which further increases the left side of the inequality.

Using the approximation of equation (2), the new specular BRDF is the following:

$$f_{r,spec}(\vec{L},\vec{V}) = P_{\vec{H}}(\vec{H}) \cdot \frac{F(\lambda,\vec{H}\cdot\vec{L})}{2(1+\vec{L}\cdot\vec{V})}. \quad (3)$$

Taking into account that the vectors have unit lengths, $2(1+\vec{L}\cdot\vec{V})$ can also be converted to the following form:

$$2(1+\vec{L}\cdot\vec{V}) = (\vec{L}\cdot\vec{L}) + 2(\vec{L}\cdot\vec{V}) + (\vec{V}\cdot\vec{V}) = (\vec{L}+\vec{V})^2 = \vec{h}\cdot\vec{h}$$

where $\vec{h}$ is the not normalized halfway vector. On the other hand, since the angle of $\vec{L}$ and $\vec{V}$ is two times the angle of $\vec{H}$ and $\vec{L}$ that was denoted by β, we can obtain:

$$2(1+\vec{L}\cdot\vec{V}) = 2(1+\cos 2\beta) = 4\cos^2\beta$$

Using these formulae, the new BRDF can also be presented in the following equivalent forms:

$$f_{r,spec}(\vec{L},\vec{V}) = P_{\vec{H}}(\vec{H}) \cdot \frac{F(\lambda,\vec{H}\cdot\vec{L})}{\vec{h}\cdot\vec{h}} = P_{\vec{H}}(\vec{H}) \cdot \frac{F(\lambda,\vec{H}\cdot\vec{L})}{4\cos^2\beta}.$$

The Fresnel function also depends on $\cos\beta$ (see appendix). Denoting $F(\lambda,\cos\beta)/(4\cos^2\beta)$ by $g_\lambda(\cos\beta)$, the BRDF is decomposed into a probability density function of $\vec{H}$ and to a single variate function $g_\lambda(\cos\beta)$ that is independent of the smoothness of the material:

$$f_{r,spec}(\vec{L},\vec{V}) = P_{\vec{H}}(\vec{H}) \cdot g_\lambda(\cos\beta).$$

For dielectric materials $g_\lambda(\cos\beta)$ can be computed by a relatively simple formula. For metals, the complex refraction index makes the computation more time consuming, but storing samples of $g_\lambda(\cos\beta)$ in one dimensional arrays the computation can be replaced by simple memory accesses.

2.1.2. Microfacet orientation probability density

Blinn[3] proposed *Gaussian distribution* for $P_{\vec{H}}(\vec{H})$, since it seemed reasonable due to the central limit value theorem of probability theory:

$$P_{\vec{H},\mathrm{Gauss}}(\vec{H}) = const \cdot e^{-(\alpha/m)^2},$$

where α is the angle of the microfacet with respect to the normal of the mean surface, that is the angle between $\vec{N}$ and $\vec{H}$, and m is the root mean square of the slope, i.e. a measure of the roughness. Later Torrance and Sparrow showed that the results of the early work of Beckmann[2] and Davies[5], who discussed the scattering of electromagnetic waves theoretically, can also be used here and thus Torrance proposed the *Beckmann distribution* function instead of the Gaussian:

$$P_{\vec{H},\mathrm{Beckmann}}(\vec{H}) = \frac{1}{m^2\cos^4\alpha} \cdot e^{-\left(\frac{\tan^2\alpha}{m^2}\right)}.$$

Unfortunately the inverse of the integral of this function cannot be obtained in closed form, thus the Beckmann distribution is not suitable for importance sampling. Let us modify this a little obtaining the following probability density:

$$P_{\vec{H},\mathrm{Ward}}(\vec{H}) = \frac{1}{m^2\pi\cos^3\alpha} \cdot e^{-\left(\frac{\tan^2\alpha}{m^2}\right)}. \qquad (4)$$

Integrating this function we get the following cumulative probability distribution, which can be inverted, thus the sampling scheme which uses two uniformly distributed numbers u,v is as follows:

$$\alpha = \arctan(m\cdot\sqrt{-\log(1-u)}), \qquad \phi = 2\pi\cdot v.$$

Note that $(1-u)$ cannot be replaced by u since random number generators produce numbers in $[0,1)$. This is basically the special case of the sampling scheme proposed by Ward[18] when isotropic reflection is assumed, thus the formula of equation (4) will be called the *Ward distribution* in this paper.

Both the Gaussian and the Beckmann distributions are bell shaped functions, which can be well approximated by the $\cos^n$ function, as happens in the Phong reflection model. Using this idea, the halfway vector can also be sampled from the following probability density[11]:

$$P_{\vec{H},\mathrm{Phong}}(\vec{H}) = \frac{(n+1)}{2\pi} \cdot \cos^n\alpha. \qquad (5)$$

Now the polar angles of the required $\vec{H}$ vector are obtained by the following transformation:

$$\alpha = \arccos u^{1/(n+1)}, \qquad \phi = 2\pi\cdot v.$$

In addition to these models, other density functions can also be used. Defining density functions by textures, for example, interesting artificial effects can be generated[1].

2.1.3. Properties of the specular component

Suppose that the Fresnel term is one, and let us examine the mirroring direction, which is the output direction if $\vec{H}=\vec{N}$. In this case $\alpha=0$ and $\beta=\theta'=\theta$, thus the BRDF is proportional to $1/\cos^2\beta = 1/\cos^2\theta$, which means that the reflected radiance at the mirroring direction increases if the incident direction goes towards 90 degrees (figure 2). This may appear as an expansion of the total volume of the reflected radiance lobe that is called the albedo (figure 3). On the other hand, since the albedo is bounded, further increase of the maximum is possible only if the reflected radiance lobe gets thinner, making the material more mirror like (figure 1).

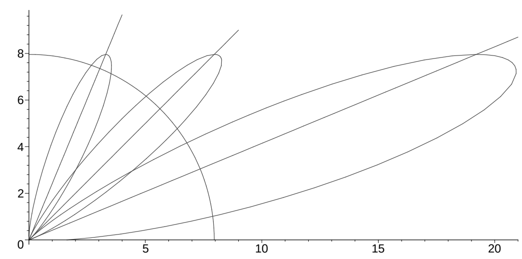

Figure 2: *Reflected radiance lobes using the Ward density for halfway vector $\vec{H}$ ($F=1, m=0.1$). The incident angles are 22.5, 45 and 67.5 degrees, respectively. Note that the maximum is below the ideal reflection direction, which is called the off-specular peak.*

2.1.4. Importance sampling of the specular component

In Monte-Carlo random walk algorithms random direction $\vec{L}$ should be obtained from a probability density $P_{\vec{L}}(\vec{L})$ that mimics $f_r(\vec{L},\vec{V})\cdot\cos\theta'$. In our case, $\vec{H}$ is sampled first using probability density $P_{\vec{H}}(\vec{H})$, then $\cos\beta$ is computed as a scalar product of $\vec{V}$ and $\vec{H}$, finally $\vec{L}$ is obtained mirroring $\vec{V}$ onto $\vec{H}$:

$$\vec{L} = 2\cos\beta\cdot\vec{H} - \vec{V}.$$

According to equation (9), this sampling scheme results in the following probability density of sampling $\vec{L}$:

$$P_{\vec{L}}(\vec{L}) = P_{\vec{H}}(\vec{H}) \cdot \frac{d\omega_{\vec{H}}}{d\omega_{\vec{L}}} = \frac{P_{\vec{H}}(\vec{H})}{4\cos\beta}.$$

If importance sampling is used, then the radiance returned from direction $\vec{L}$ is weighted by

$$\frac{f_r(\vec{L},\vec{V}) \cdot \cos\theta'}{P_{\vec{L}}(\vec{L})} = F(\lambda, \cos\beta) \cdot \frac{\cos\theta'}{\cos\beta}.$$

The quality of importance, i.e. BRDF sampling is determined by the variation of this weight. If it is close to constant, then the BRDF sampling is effective. Optimal sampling, when the weight is constant, is unfortunately possible only for simple BRDFs such as for the diffuse BRDF or the ideal mirror BRDF. For example, when the reciprocal Phong model is sampled, the weight is not constant but is the product of $\cos\theta'$ and a factor determining that portion of the lobe which is not cut off by the surface [11, 16].

For the new model, if the Fresnel function is close to 1 and the surface is really specular, i.e. the random halfway vector is close the normal vector, then $\beta \approx \theta'$, thus the weight is almost 1, which means a nearly optimal importance sampling. Note that $\cos\theta'/\cos\beta$ is less than 2 even in the worst case. For more specular cases, this ratio is very close to 1. This means that the new model has more effective BRDF sampling than that of the Phong formula.

2.1.5. Albedo of the specular component

The albedo of the specular component can be obtained as an integral of the cosine weighted BRDF:

$$a_{spec}(\theta') = a_{spec}(\vec{L}) = \int_{\Omega} f_{r,spec}(\vec{L},\vec{V}) \cdot \cos\theta \, d\omega_{\vec{V}}.$$

This integral can be evaluated by a Monte-Carlo quadrature that applies the proposed importance sampling:

$$a_{spec}(\vec{L}) = \int_{\Omega_{\vec{H}}} P_{\vec{H}}(\vec{H}) \cdot \frac{F(\lambda,\cos\beta)\cos\theta}{\cos\beta} \, d\omega_{\vec{H}} =$$

$$E\left[\frac{F(\lambda,\cos\beta)\cos\theta}{\cos\beta}\right] \approx \sum_i \frac{F(\lambda,\cos\beta_i)\cos\theta_i}{\cos\beta_i}$$

where halfway vector samples should be generated according to density $P_{\vec{H}}(\vec{H})$. Due to the effective importance sampling, accurate albedo values can be obtained from a few samples. In a preprocessing phase the albedo curves of all materials present in the scene are computed and the results are stored in one-dimensional tables.

Figure 3 shows the albedo function assuming different microfacet densities. The correspondence between the exponent n of the Phong model and the roughness m of the Ward distribution was established to make the maximum of the lobes equal at perpendicular illumination.

2.2. Coupling the matte and the specular components

The complete BRDF model is a sum of the proposed specular component $f_{r,spec}(\vec{L},\vec{V})$ which accounts for single microfacet reflections and a matte component $f_{r,matte}(\vec{L},\vec{V})$ which represents the multiple reflections and the re-emissions of absorbed photons:

$$f_r(\vec{L},\vec{V}) = f_{r,spec}(\vec{L},\vec{V}) + f_{r,matte}(\vec{L},\vec{V}).$$

It would be very difficult to elaborate a probabilistic model for the phenomena described by the matte component, thus a simpler empirical approach is followed keeping in mind the requirements of symmetry and energy conservation. To guarantee the symmetry, the matte BRDF is searched in symmetric separable form [6, 14, 1]:

$$f_{r,matte}(\vec{L},\vec{V}) = k(\lambda) \cdot s \cdot r(\theta) \cdot r(\theta'),$$

where $k(\lambda)$ is a wavelength dependent color variable that is between 0 and 1, s is a normalization constant that is responsible for limiting the albedo to 1, and r is an appropriate scalar function. The albedo of such material is:

$$a_{matte}(\theta') = k(\lambda) \cdot s \cdot 2\pi \cdot r(\theta') \cdot \int_0^{\pi/2} r(\theta)\cos\theta\sin\theta \, d\theta. \quad (6)$$

Note that the albedo is proportional to $r(\theta')$. Concerning energy conservation the total albedo, including both the specular and the matte components, cannot exceed 1, thus $a_{matte}(\theta')$ cannot be greater than $1 - a_{spec}(\theta')$. In the perfectly reflecting case $k(\lambda)$ and the total albedo is 1, thus the albedo of the matte component is:

$$a_{matte}(\theta') = 1 - a_{spec}(\theta'). \quad (7)$$

As concluded, function $r(\theta')$ is proportional to the albedo, thus it is also proportional to $1 - a_{spec}(\theta')$, which gives the following formulae for the perfectly reflecting case:

$$f_{r,matte}(\vec{L},\vec{V}) = s \cdot (1 - a_{spec}(\theta')) \cdot (1 - a_{spec}(\theta)).$$

The normalization constant s can be obtained by substituting this BRDF into equations (6) and (7), when we can obtain:

$$s = \frac{1}{\pi(1 - a_{spec}^{ave})},$$

where $a_{spec}^{ave} = \frac{1}{\pi} \cdot \int_{\Omega} a_{spec}(\theta)\cos\theta \, d\omega_{\vec{V}}$ is the *average albedo* of the specular component. Summarizing, the BRDF of the matte component is as follows:

$$f_{r,matte}(\vec{L},\vec{V}) = k(\lambda) \cdot \frac{(1 - a_{spec}(\theta')) \cdot (1 - a_{spec}(\theta))}{\pi(1 - a_{spec}^{ave})}.$$

2.2.1. Importance sampling of the matte component

Since the specular albedo appearing in the matte BRDF formula is stored in tables, importance sampling, which consists of the integration of the cosine weighted BRDF and the

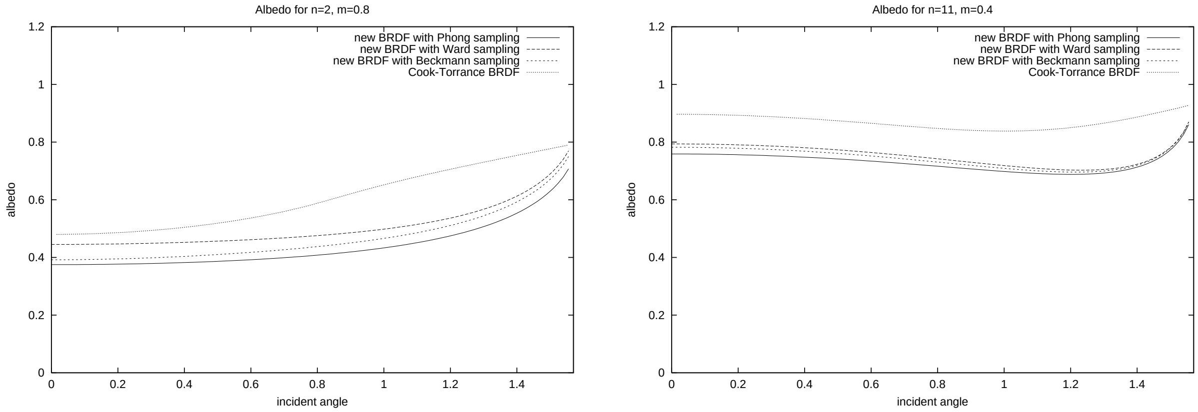

Figure 3: *Albedo curves of the specular component with different microfacet orientation probabilities*

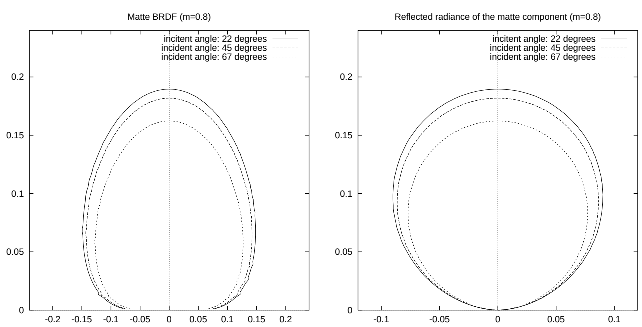

Figure 4: *Lobes of the matte BRDF (left) and the of the corresponding reflected radiance (right) for different incident angles. Wards halfway vector density is used with* $m = 0.8$.

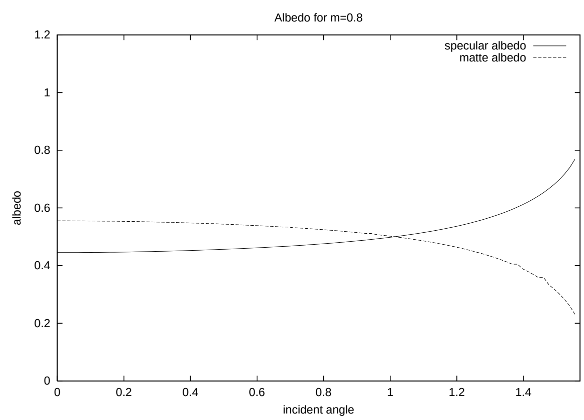

Figure 5: *Albedo curves of the specular and matte components for* $m = 0.8$*. The total albedo is 1.*

inversion of the result, cannot be an analytic process. However, since the matte component depends just on the incident and viewing angles and is in separable form, the problem can be solved using a single one-dimensional table. Let us define the cumulative reflection function as follows:

$$I(\theta') = \int_0^{\theta'} (1 - a_{spec}(\xi)) \cdot \cos\xi \, d\xi.$$

As can be shown easily, if u is a uniformly distributed variable in $[0, 1)$, then the probability density of

$$\theta' = I^{-1}\left(u \cdot I\left(\frac{\pi}{2}\right)\right)$$

will be proportional to $(1 - a_{spec}(\theta')) \cdot \cos\theta'$, which is required by the exact importance sampling. This inverse cumulative reflection function can be stored in a table, which is accessed when a new direction is needed.

The complete BRDF can be sampled according to the principles of Russian roulette, which selects randomly from the possibilities of sampling the specular BRDF, sampling the matte component, or terminating the walk. If the arrival angle is θ, the specular BRDF is selected with $a_{spec}(\theta)$ probability, the matte BRDF is with $\tilde{k} \cdot (1 - a_{spec}(\theta))$ probability, where $\tilde{k}$ is the average of the color components. The walk is stopped with $(1 - \tilde{k})(1 - a_{spec}(\theta))$ probability.

3. Simulation results

The proposed BRDF model has been implemented in a Monte-Carlo rendering system that uses bi-directional ray-tracing[10].

Figure 6 compares the original Cook-Torrance model with the new BRDF and demonstrates that although the new model is simpler and meets the requirements of importance sampling, the visual quality is similar to that of the Cook-Torrance BRDF. Figure 7 shows scenes containing plastic materials that are simulated by the coupled specular-matte model. We also examined real billiard balls and noticed that the proposed model accurately mimics the reflection that can be observed in real life, for example, the balls similarly turn whiter at grazing angles. In order to present the dynamic coupling between the specular and matte components, figure 8 shows the same scene from different viewing angles. Note that the base surface becomes more mirror like at larger viewing angles, similarly to the real table in figure 1.

4. Conclusions

This paper presented a physically plausible coupled BRDF model. The specular part is physically based and is derived by the simplification of the results obtained by the analysis of the microfacet reflection model. The simplifications have only slightly modified the appearance of the materials, but significantly simplified the computations and made importance sampling possible. The specular part provides those important features that can be observed in real world, as for example, it exhibits off-specular peak and becomes mirror-like towards grazing angles.

The specular component is complemented by a matte component which is not independent of the specular part, but the two reflection modes are coupled. Such coupling is also a real-world phenomenon, but has usually been ignored in computer graphics. This paper generalized the previous work, and proposed a scheme coupling the matte and the non-ideal specular reflection, which is also good for importance sampling. The new matte component is also physically plausible, but is not physically based.

Importance sampling has been proposed for both the specular and the matte component. We concluded that the quality of BRDF sampling is even better than that of the Phong model. The proposed importance sampling scheme and coupling can also be used with other BRDF models.

5. Acknowledgements

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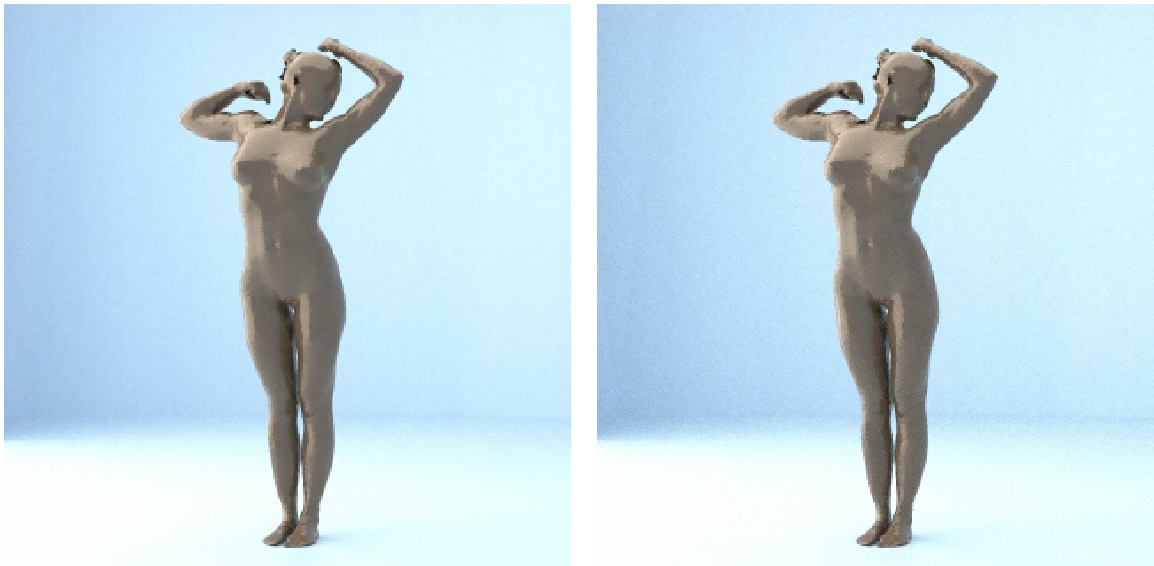

Figure 6: *Comparison of the Cook-Torrance (left) and the new models (right). The girl is illuminated by large light sources and is standing in a* 0.8 *albedo environment. The model is from 3D Cafe (http:// www.3dcafe.com)*

Figure 7: *Scenes of plastic Buddha and balls. The index of refraction used by the Fresnel function is 1.7 and the roughness parameter m is 0.03. The Buddha model is from the Stanford 3D Scanning Repository (http://graphics.stanford.edu/data/3Dscanrep/)*

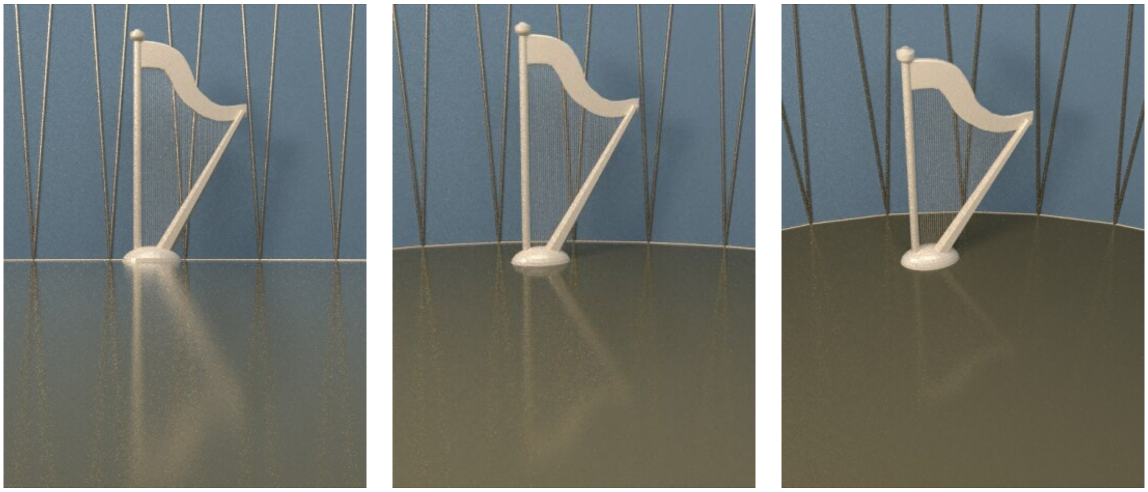

Figure 8: *Coupling of specular and matte components for different viewing angles*

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Appendix: A probabilistic derivation of the Cook-Torrance model

Concerning the specular reflection from direction $\vec{L}$ to $d\omega_{\vec{V}}$ around direction $\vec{V}$, only those facets can contribute whose normal is in $d\omega_{\vec{H}}$ around the halfway unit vector $\vec{H}$. Suppose that all facets have size S. If reflection is to happen, the facet should not be hidden by other facets, nor should its reflection run into other facets, and it should not absorb the photon. Considering these facts, the event that "a photon is reflected directly to $d\omega_{\vec{V}}$ around $\vec{V}$" can be expressed as the logical AND connection of the following events:

1. **Orientation**: In the path of the photon there is a microfacet having its normal in $d\omega_{\vec{H}}$ around $\vec{H}$.
2. **No shadowing or masking**: The given microfacet is not hidden by other microfacets from the photon coming from the light source, and the reflected photon does not run into another microfacet.
3. **Reflection**: The photon is not absorbed by the microfacet that is supposed to be a perfect mirror.

Since the reflection from an ideal mirror microfacet and the shadowing and masking are stochastically independent if the orientation is given, the probability of the following composed event can be expressed as

$$\Pr\{\text{orientation} \wedge \text{no mask and shadow} \wedge \text{reflection}\} =$$

$$\Pr\{\text{no mask and shadow} \mid \text{orientation}\}\cdot$$

$$\Pr\{\text{reflection} \mid \text{orientation}\} \cdot \Pr\{\text{orientation}\}.$$

If a photon arrives from direction $\vec{L}$ to a surface element dA, the visible area of the surface element will be $dA \cdot (\vec{N} \cdot \vec{L})$, while the total visible area of the microfacets having their normal in the direction around $\vec{H}$ will be the product of the visible size of a single microfacet $(S \cdot (\vec{H} \cdot \vec{L}))$, and the number of properly oriented microfacets, which, in turn, is the product of the probability that a microfacet is properly oriented $(P_{\vec{H}}(\vec{H}) \cdot d\omega_{\vec{H}})$ and the number of all microfacets (dA/S). The probability of finding an appropriate microfacet aligned with the photon is the visible size of properly oriented microfacets divided by the visible surface area, thus we obtain:

$$\Pr\{\text{orientation}\} = \frac{S \cdot (\vec{H} \cdot \vec{L}) \cdot P_{\vec{H}}(\vec{H}) \cdot d\omega_H \cdot dA/S}{dA \cdot (\vec{N} \cdot \vec{L})}.$$

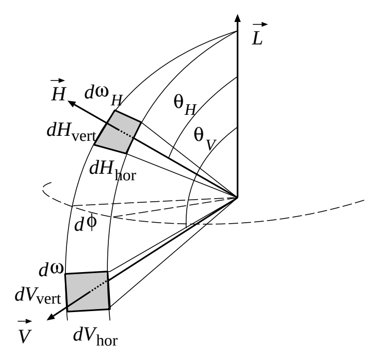

Figure 9: *Calculation of* $d\omega_{\vec{H}}/d\omega_{\vec{V}}$

Unfortunately, in the BRDF the transfer probability density uses $d\omega_{\vec{V}}$ instead of $d\omega_{\vec{H}}$, thus we have to determine $d\omega_{\vec{H}}/d\omega_{\vec{V}}$ [8]. Defining a spherical coordinate system (θ,ϕ), with the north pole in the direction of $\vec{L}$ (figure 9), the solid angles are expressed by the product of vertical and horizontal arcs:

$$d\omega_{\vec{V}} = dV_{hor}\cdot dV_{vert}, \quad d\omega_{\vec{H}} = dH_{hor}\cdot dH_{vert}.$$

In this spherical coordinate system $\theta_H = \beta$ and $\theta_V = 2\beta$. Using geometric considerations, we get:

$$dV_{hor} = d\phi\cdot\sin\theta_V, \quad dH_{hor} = d\phi\cdot\sin\theta_H, \quad dV_{vert} = 2dH_{vert},$$

which in turn yields:

$$\frac{d\omega_{\vec{H}}}{d\omega_{\vec{V}}} = \frac{\sin\theta_H}{2\sin\theta_V} = \frac{\sin\beta}{2\sin 2\beta} = \frac{1}{4\cos\beta} = \frac{1}{4(\vec{L}\cdot\vec{H})}. \quad (8)$$

If view vector $\vec{V}$ is fixed, then the same reasoning results in:

$$\frac{d\omega_{\vec{H}}}{d\omega_{\vec{L}}} = \frac{1}{4\cos\beta}. \quad (9)$$

Thus the orientation probability is

$$\Pr\{\text{orientation}\} = \frac{P_{\vec{H}}(\vec{H})}{4(\vec{N}\cdot\vec{L})}\cdot d\omega_{\vec{V}}.$$

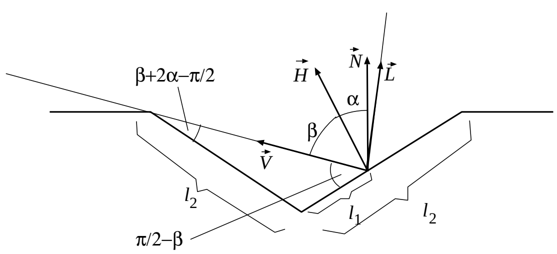

Figure 10: *Geometry of masking*

The visibility of the microfacets from direction $\vec{V}$ means that the reflected photon does not run into another microfacet. The collision is often referred to as *masking*. Looking at figure 10, we can easily recognize that the probability of masking is l_1/l_2, where l_2 is the one-dimensional length of the microfacet, and l_1 describes the boundary case from where the beam is masked. The angles of the triangle formed by the bottom of the microfacet wedge and the beam in the boundary case can be expressed by the angles $\alpha = \text{angle}(\vec{N},\vec{H})$ and $\beta = \text{angle}(\vec{V},\vec{H}) = \text{angle}(\vec{L},\vec{H})$ by geometric considerations and by using the law of reflection. Applying the sine law for this triangle, and some trigonometric formulae:

$$\Pr\{\text{not masking}|\text{ orientation}\} = 1 - \frac{l_1}{l_2} =$$

$$1 - \frac{\sin(\beta+2\alpha-\pi/2)}{\sin(\pi/2-\beta)} = 2\cdot\frac{\cos\alpha\cdot\cos(\beta+\alpha)}{\cos\beta}.$$

According to the definitions of the angles $\cos\alpha = \vec{N}\cdot\vec{H}$, $\cos(\beta+\alpha) = \vec{N}\cdot\vec{V}$ and $\cos\beta = \vec{V}\cdot\vec{H}$.

If the angle of incident light and the facet normal do not allow the triangle to be formed, the probability of no masking taking place is obviously 1. This situation can be recognized by evaluating the formula without any previous considerations and checking whether the result is greater than 1, then limiting the result to 1. The final result is:

$$\Pr\{\text{not masking}|\text{ orientation}\} = \min\{2\cdot\frac{(\vec{N}\cdot\vec{H})\cdot(\vec{N}\cdot\vec{V})}{(\vec{V}\cdot\vec{H})}, 1\}.$$

The probability of *shadowing* can be derived in exactly the same way, only $\vec{L}$ should be substituted with $\vec{V}$. Since for a given microfacet orientation and light direction, either masking or shadowing implies the other, the probability of neither shadowing nor masking taking place can be obtained by the minimum of the two probabilities:

$$\Pr\{\text{no shadow and mask}|\text{ orientation}\} =$$

$$\min\{2\cdot\frac{(\vec{N}\cdot\vec{H})\cdot(\vec{N}\cdot\vec{V})}{(\vec{V}\cdot\vec{H})}, 2\cdot\frac{(\vec{N}\cdot\vec{H})\cdot(\vec{N}\cdot\vec{L})}{(\vec{L}\cdot\vec{H})}, 1\} = G(\vec{N},\vec{L},\vec{V}).$$

Even a perfect mirror absorbs some portion of the incident light, as is described by the *Fresnel equations.* Since $F(\lambda,\theta')$ is the fraction of the reflected energy, it also describes the probability of a photon being reflected at a microfacet that is assumed to be an ideal mirror, giving:

$$\Pr\{\text{reflection}|\text{ orientation}\} = F(\lambda,\vec{H}\cdot\vec{L})$$

where variable θ' has been replaced by $\vec{H}\cdot\vec{L} = \cos\beta$ since those microfacets that reflect from $\vec{L}$ to $\vec{V}$ have normal vector equal to $\vec{H}$.

Now we can summarize the results by multiplying the probabilities to express $w(\vec{L},\vec{V})d\omega_{\vec{V}}$:

$$w(\vec{L},\vec{V})d\omega_{\vec{V}} = \frac{P_{\vec{H}}(\vec{H})}{4(\vec{N}\cdot\vec{L})}\cdot G(\vec{N},\vec{L},\vec{V})\cdot F(\lambda,\vec{H}\cdot\vec{L})\ d\omega_{\vec{V}}.$$

The BRDF is the reflection density divided by the cosine of the outgoing angle $(\vec{N}\cdot\vec{V})$, thus we obtain

$$f_r(\vec{L},\vec{V}) = \frac{P_{\vec{H}}(\vec{H})}{4(\vec{N}\cdot\vec{L})(\vec{N}\cdot\vec{V})}\cdot G(\vec{N},\vec{L},\vec{V})\cdot F(\lambda,\vec{H}\cdot\vec{L}).$$

If $P_{\vec{H}}(\vec{H})$ is the Beckmann distribution, this formula becomes the famous Cook-Torrance model.